Chapter 11

Introduction to Algebra
Chapter Outline

- Introduction to Variables
- Solving Equations: The Addition Property
- Solving Equations: The Multiplication Property
- Solving Equations Using Addition and Multiplication Properties
- Equations and Problem Solving
§ 11.1

Introduction to Variables
Section Objectives

- Evaluating Algebraic Expressions
- Combining Like Terms
- Multiplying Expressions
## Evaluating Algebraic Expressions

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable</strong>: A letter to represent all the numbers fitting a pattern.</td>
<td>Evaluate: $7 + 3z$ when $z = -3$</td>
</tr>
<tr>
<td><strong>Algebraic Expression</strong>: A combination of numbers, letters (variables), and operation symbols.</td>
<td><strong>SOLUTION</strong></td>
</tr>
<tr>
<td><strong>Evaluating the Expression</strong>: Replacing a variable in an expression by a number and then finding the value of the expression.</td>
<td>$7 + 3(-3)$ = $7 + (-9)$ = $7 - 9$ = $-2$</td>
</tr>
</tbody>
</table>
## Combining Like Terms

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
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</thead>
</table>
| **Terms**: The addends of an algebraic expression. | $2x + x = 3x$  
$x \cdot x = x^2$  
$2x \neq x^2$ |
| **Constant Term**: A term that is only a number. | $3x + 2$   |
| **Variable Term**: A term that contains a variable. | $4x + 5$   |
| **Numerical Coefficient**: The number factor of a variable term. | $7x + 1$   |
## Combining Like Terms

<table>
<thead>
<tr>
<th>Distributive Property</th>
<th>Properties of Addition and Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $a$, $b$, and $c$ are numbers, then $ac + bc = (a + b)c$</td>
<td>If $a$, $b$, and $c$ are numbers, then $a + b = b + a$</td>
</tr>
<tr>
<td>Also, $ac - bc = (a - b)c$</td>
<td>Also, $a \cdot b = b \cdot a$</td>
</tr>
<tr>
<td></td>
<td>(a + b) + c = a + (b + c)</td>
</tr>
<tr>
<td></td>
<td>(a \cdot b) \cdot c = a \cdot (b \cdot c)</td>
</tr>
</tbody>
</table>

### EXAMPLE
Simplify $8y + 3y$.

### SOLUTION

\[ 8y + 3y = (8 + 3)y \]

\[ = 11y \]
EXAMPLE

Multiply:  $4(5y-6)$

SOLUTION

$$4(5y - 6) = 4(5y) - 4(6) = 20y - 24$$
§ 11.2

Solving Equations: The Addition Property
Section Objectives

- Determining Whether a Number is a Solution
- Using the Addition Property to Solve Equations
## Determining Whether a Number is a Solution

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solving</strong>: In an equation containing a variable, finding which values of the variable make the equation a true statement.</td>
<td>Is -7 a solution of: ( a + 23 = -16 )?</td>
</tr>
</tbody>
</table>
| **Solution**: In an equation, a value for the variable that makes the equation a true statement. | **SOLUTION**  
\[
\begin{align*}
  a + 23 &= -16 \\
  (-7) + 23 &\neq -16
\end{align*}
\]  
-7 is not a solution of \( a \). |
Using the Addition Property to Solve Equations

**Distributive Property**

Let a, b, and c represent numbers. Then

\[
\begin{align*}
    a &= b \\
    \text{and } a + c &= b + c \\
    \text{are equivalent equations}
\end{align*}
\]

Also,

\[
\begin{align*}
    a &= b \\
    \text{and } a - c &= b - c \\
    \text{are equivalent equations}
\end{align*}
\]

**EXAMPLE**

Solve: \( f + 4 = -6 \)

**SOLUTION**

\[
\begin{align*}
    f + 4 &= -6 \\
    (f + 4) - 4 &= (-6) - 4 \\
    f + 4 - 4 &= -6 - 4 \\
    f + 0 &= -10 \\
    f &= -10
\end{align*}
\]
§ 11.3

Solving Equations: The Multiplication Property
Section Objectives

- Using the Multiplication Property to Solve Equations
Using the Multiplication Property to Solve Equations

### Multiplication Property of Equality

Let $a$, $b$, and $c$ represent numbers and $c \neq 0$. Then

- $a = b$  
- and $a \cdot c = b \cdot c$

are equivalent equations

Also, $a = b$

\[
\frac{a}{c} = \frac{b}{c}
\]

are equivalent equations

---

**EXAMPLE**

Solve: $7y = 21$.

**SOLUTION**

\[
\begin{align*}
7y &= 21 \\
\frac{7y}{7} &= \frac{21}{7} \\
y &= 3
\end{align*}
\]
§ 11.4

Solving Equations Using Addition and Multiplication Properties
Section Objectives

- Solving Equations Using Addition and Multiplication Properties
- Solving Equations Containing Parentheses
- Writing Sentences as Equations
EXAMPLE
Solve: \(3y - 12 = 0\).

SOLUTION

\[
\begin{align*}
3y - 12 &= 0 \\
3y - 12 + 12 &= 0 + 12 \\
3y &= 12 \\
\frac{3y}{3} &= \frac{12}{3} \\
y &= 4
\end{align*}
\]
**Steps** | **Example**
--- | ---
*Step 1:* If parentheses are present, use the distributive property.  
Solve: \(3(x-1) = 12\)  
*Step 1:* \(3(x-1) = 12\)  
\(3(x) - 3(1) = 12\)  

*Step 2:* Combine any like terms on each side of the equation.  
*Step 2:* \(3x - 3 = 12\)  

*Step 3:* Use the addition property of equality to rewrite the equation so that variable terms are on one side of the equation and constant terms are on the other side.  
*Step 3:* \(3x - 3 + 3 = 12 + 3\)  
\(3x = 15\)  

*Step 4:* Use the multiplication property of equality to divide both sides by the numerical coefficient of the variable to solve for.  
*Step 4:* \(\frac{3x}{3} = \frac{15}{3}\)  
\(x = 5\)  

*Step 5:* Check the solution in the original equation.  
*Step 5:* \(3(x - 1) = 12\)  
\(3(5 - 1) = 12\)
# Writing Sentences as Equations

<table>
<thead>
<tr>
<th>Key Words or Phrases</th>
<th>Examples</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>equals</td>
<td>3 equals 2 plus 1</td>
<td>$3 = 2 + 1$</td>
</tr>
<tr>
<td>gives</td>
<td>the quotient of 10 and -5 gives -2</td>
<td>$\frac{10}{-5} = -2$</td>
</tr>
<tr>
<td>is/was</td>
<td>17 minus 12 is 5</td>
<td>$17 - 12 = 5$</td>
</tr>
<tr>
<td>yields</td>
<td>11 plus 2 yields 13</td>
<td>$11 + 2 = 13$</td>
</tr>
<tr>
<td>amounts to</td>
<td>twice -15 amounts to -30</td>
<td>$2(-15) = -30$</td>
</tr>
<tr>
<td>is equal to</td>
<td>-24 is equal to 2 times -12</td>
<td>$-24 = 2(-12)$</td>
</tr>
</tbody>
</table>
EXAMPLE
Write the following sentence as an equation:

The product of \(-5\) and \(-29\) gives 145.

SOLUTION

\((-5) \cdot (-29) = 145\)
§ 11.5

Equations and Problem Solving
Section Objectives

- Writing Phrases as Algebraic Expressions
- Writing Sentences as Equations
- Using Problem-Solving Steps to Solve Problems
Writing Phrases as Algebraic Expressions

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
<th>Equal Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
<td>equals</td>
</tr>
<tr>
<td>plus</td>
<td>minus</td>
<td>times</td>
<td>divided by</td>
<td>gives</td>
</tr>
<tr>
<td>added to</td>
<td>subtracted from</td>
<td>multiply</td>
<td>into</td>
<td>is/was</td>
</tr>
<tr>
<td>more than</td>
<td>less than</td>
<td>twice</td>
<td>per</td>
<td>yields</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>of</td>
<td></td>
<td>amounts to</td>
</tr>
<tr>
<td>total</td>
<td>less</td>
<td>double</td>
<td></td>
<td>is equal to</td>
</tr>
</tbody>
</table>
EXAMPLE
Write the following phrase as a variable expression:
Ten plus a number

SOLUTION
$10 + x$
Writing Sentences as Equations

**EXAMPLE**
Write the following sentence as an equation:

Five subtracted from a number equals 10.

**SOLUTION**

\[ x - 5 = 10 \]
Using Problem-Solving Steps to Solve Problems

Steps

1. UNDERSTAND the problem.

2. TRANSLATE THE PROBLEM INTO AN EQUATION.

3. SOLVE the equation.

4. INTERPRET the results.
EXAMPLE
The sum of 7, 9, and a number is 40. Find the number.  

SOLUTION

\[7 + 9 + x = 40\]  \hspace{1cm}\text{UNDERSTAND}

\[
(16) + x = 40 \hspace{1cm} \text{TRANSLATE}
\]

\[16 + x - 16 = 40 - 16\]

\[x = 24\]

\[7 + 9 + x = 40\]  \hspace{1cm}\text{INTERPRET}

\[(16) + 24 = 40\]

The unknown number is 24.