Chapter 3

Adding and Subtracting Fractions
Chapter Outline

- Adding and Subtracting Like Fractions
- Least Common Multiple
- Adding and Subtracting Unlike Fractions
- Adding and Subtracting Mixed Numbers
- Order, Exponents, and the Order of Operations
- Fractions and Problem Solving
§ 3.1

Adding and Subtracting Like Fractions
Section Objectives

- Adding Like Fractions
- Subtracting Like Fractions
- Solving Problems by Adding or Subtracting Like Fractions
## Adding Like Fractions

### Formula

\[
\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}
\]

### Example

\[
\frac{9}{17} + \frac{2}{17} = \frac{9 + 2}{17} = \frac{11}{17}
\]
# Subtracting Like Fractions

The formula for subtracting like fractions is:

\[ \frac{a}{c} - \frac{b}{c} = \frac{a - b}{c} \]

**Example**

\[ \frac{9}{13} + \frac{5}{13} = \frac{9 + 5}{13} = \frac{14}{13} \]
Solving Problems by Adding or Subtracting Like Fractions

EXAMPLE

Find the perimeter of the following rectangle.

\[
\text{Perimeter} = \frac{5}{12} + \frac{5}{12} + \frac{7}{12} + \frac{7}{12} = \frac{5+5+7+7}{12}
\]

\[
= \frac{24}{12} = 2 \text{ m}^2
\]
§ 3.2

Least Common Multiple
Section Objectives

- Finding the Least Common Multiple Using Multiples
- Finding the LCM Using Prime Factorization
- Writing Equivalent Fractions
Finding the Least Common Multiple Using Multiples

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Least Common Multiple (LCM):</strong></td>
<td>Multiples of 4: 4, 8, 12, 16, 20, 24, 28, …</td>
</tr>
<tr>
<td>The smallest number in the list of common multiples.</td>
<td>Multiples of 6: 6, 12, 18, 24, 30, 36, 42, …</td>
</tr>
</tbody>
</table>
Finding the LCM Using Prime Factorization

<table>
<thead>
<tr>
<th>Step</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Step 1**: Write the prime factorization of each number. | Prime factorization for 15: 3, 5  
Prime factorization for 20: 2, 2, 5 |
| **Step 2**: For each different prime factor in step 1, circle the greatest number of times that factor occurs in any one factorization. (Do not circle the same numbers in both prime factorizations.) | Prime factorization for 15: 3, 5  
Prime factorization for 20: 2, 2, 5 |
| **Step 3**: The LCM is the product of the circled factors. | LCM = 2·2·3·5 = 60 |
Writing Equivalent Fractions

EXAMPLE
Write \( \frac{3}{5} = \frac{?}{20} \) as an equivalent fraction.

SOLUTION
Multiply \( \frac{3}{5} \) by 1 in the form of \( \frac{4}{4} \) to create a common denominator of 20.

\[
\frac{3 \cdot 4}{5 \cdot 4} = \frac{3 \cdot 4}{5 \cdot 4} = \frac{12}{20}
\]

Therefore \( \frac{3}{5} = \frac{?}{20} = \frac{3}{5} = \frac{12}{20} \)
§ 3.3

Adding and Subtracting Unlike Fractions
Section Objectives

- Adding Unlike Fractions
- Subtracting Unlike Fractions
- Solving Problems by Adding or Subtracting Unlike Fractions
## Adding Unlike Fractions

<table>
<thead>
<tr>
<th>Step</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Step 1**: Find the LCM of the denominators of the fractions. This number is the least common denominator (LCD). | \[
\frac{1}{2} + \frac{1}{3} \quad \text{Multiples of 2: 2, 4, 6, 8, 10, \ldots} \\
\frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2} = \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} = \frac{3}{6} + \frac{2}{6} \\
\frac{3}{6} + \frac{2}{6} = \frac{5}{6} = \frac{5}{6} \\
\frac{5}{6} \quad \text{is in its simplest form.}|

<table>
<thead>
<tr>
<th><strong>Step 2</strong>: Write each fraction as an equivalent fraction whose denominator is the LCD.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 3</strong>: Add or subtract the like fractions.</td>
<td></td>
</tr>
<tr>
<td><strong>Step 4</strong>: Write the sum or difference in simplest form.</td>
<td></td>
</tr>
</tbody>
</table>
Subtracting Unlike Fractions

**EXAMPLE**

Subtract and simplify: \( \frac{3}{4} - \frac{1}{7} \)

**SOLUTION**

**Step 1:** The LCD of 4 and 7 is 28

\[
\frac{3}{4} - \frac{1}{7} = \frac{3 \cdot 7}{4 \cdot 7} - \frac{1 \cdot 4}{7 \cdot 4} = \frac{21}{28} - \frac{4}{28}
\]

**Step 2:**

\[
\frac{21}{28} - \frac{4}{28} = \frac{21 - 4}{28} = \frac{17}{28}
\]

**Step 4:** \( \frac{17}{28} \) is in its simplest form.
Solving Problems by Adding or Subtracting Unlike Fractions

**EXAMPLE**

The slowest mammal is the three-toed sloth from South America. The sloth has an average ground speed of \( \frac{1}{10} \) mph. In the trees, it can accelerate to \( \frac{17}{100} \) mph. How much faster can a sloth travel in the trees? *(Source: The Guiness Book of World Records)*

**SOLUTION**

*Step 1:* The LCD of 10 and 100 is 100

*Step 2:* \[
\frac{17}{100} - \frac{1}{10} = \frac{17}{100} - \frac{1 \cdot 10}{10 \cdot 10} = \frac{17 - 10}{100}
\]

*Step 3:* \[
\frac{17}{100} - \frac{10}{100} = \frac{17-10}{100} = \frac{7}{100}
\]

*Step 4:* \( \frac{7}{100} \) mph is in its simplest form.
§ 3.4

Adding and Subtracting Mixed Numbers
Section Objectives

- Adding Mixed Numbers
- Subtracting Mixed Numbers
- Solving Problems by Adding or Subtracting Mixed Numbers
Adding Mixed Numbers

**EXAMPLE**

Add: \(12 \frac{5}{12} + 4 \frac{1}{6}\)

**SOLUTION**

The LCD of 6 and 12 is 12.

\[
\begin{align*}
12 \frac{5}{12} & \quad 12 \frac{5}{12} \\
+ 4 \frac{1}{6} & \quad + 4 \frac{2}{12} \\
+ 6 & \quad + 6 \\
\hline
16 \frac{7}{12} & \\
\end{align*}
\]

Add the fractions. Then add the whole numbers.
Subtracting Mixed Numbers

**EXAMPLE**

Subtract: \[ 9 \frac{1}{5} \]
\[-8 \frac{6}{25} \]

**SOLUTION**

The LCD of 5 and 25 is 25.

\[ \frac{9}{5} - \frac{6}{25} = \frac{45}{25} - \frac{6}{25} = \frac{39}{25} \]

Because 5 is less than 6, we need to borrow 1 from 9.

Subtract the fractions.

Then subtract the whole numbers.
Solving Problems by Adding or Subtracting Mixed Numbers

EXAMPLE

If the total weight allowable without overweight charges is 50 pounds and the traveler’s luggage weighs 60\(\frac{5}{8}\) pounds, on how many pounds will the traveler’s overweight charges be based?

SOLUTION

\[
\begin{align*}
60\frac{5}{8} & \quad -50\frac{0}{8} \\
\hline
10\frac{5}{8} & \\
\end{align*}
\]

Subtract the fractions. Then subtract the whole numbers.

The traveler’s overweight charges will be based on 10\(\frac{5}{8}\) lbs.
§ 3.5

Order, Exponents, and the Order of Operations
Section Objectives

- Comparing Fractions
- Evaluating Fractions Raised to Powers
- Reviewing Operations on Fractions
- Using the Order of Operations
Comparing Fractions

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inequality Symbols:</strong></td>
<td></td>
</tr>
<tr>
<td>(&lt;) means less than</td>
<td>9 (&gt;) 3</td>
</tr>
<tr>
<td>(&gt;) means greater than</td>
<td></td>
</tr>
<tr>
<td><strong>Comparing Fractions:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Step 1:</strong> Write the fractions as like fractions</td>
<td>(\frac{5}{8} &gt; \frac{14}{40})</td>
</tr>
<tr>
<td><strong>Step 2:</strong> The fraction with the greater numerator is the greater fraction.</td>
<td>(\frac{15}{40} &gt; \frac{14}{40})</td>
</tr>
</tbody>
</table>
Evaluating Fractions Raised to Powers

EXAMPLE
Evaluate \( \left( \frac{1}{7} \right)^2 \).

SOLUTION

\[
\left( \frac{1}{7} \right)^2 = \frac{1}{7} \cdot \frac{1}{7} = \frac{1 \cdot 1}{7 \cdot 7} = \frac{1}{49}
\]
### Review of Operations on Fractions

<table>
<thead>
<tr>
<th>Operation</th>
<th>Procedure</th>
<th>Example</th>
</tr>
</thead>
</table>
| Multiply        | Multiply the numerators and multiply the denominators                     | \[
\frac{3 \cdot 1}{7 \cdot 5} = \frac{3}{35}
\] |
| Divide          | Multiply the first fraction by the reciprocal of the second fraction      | \[
\frac{9}{10} \div \frac{2}{3} = \frac{9}{10} \cdot \frac{3}{2} = \frac{9 \cdot 3}{10 \cdot 2} = \frac{27}{20}
\] |
| Add or Subtract | 1. Write each fraction as an equivalent fraction whose denominator is the LCD  
               | 2. Add or subtract numerators and write the result over the common denominator. | \[
\frac{5}{12} + \frac{5}{6} = \frac{5}{12} + \frac{5 \cdot 2}{6 \cdot 2} = \frac{5}{12} + \frac{10}{12} = \frac{15}{12}
\]
|                 |                                                                           | \[
\frac{1}{9} - \frac{4}{9} = \frac{9}{9} - \frac{4}{9} = \frac{5}{9}
\] |
Order of Operations

1. Perform all operations within parentheses ( ), brackets [ ], or other grouping symbols such as fraction bars or square roots.

2. Evaluate any expressions with exponents.

3. Multiply or divide in order from left to right.

4. Add or subtract in order from left to right.
Using Order of Operations

**EXAMPLE**

Use the order of operations to simplify \( \frac{1}{2} + \frac{1 \cdot 1}{6 \cdot 3} \).

**SOLUTION**

\[
\begin{align*}
\frac{1}{2} + \frac{1 \cdot 1}{6 \cdot 3} & \quad \text{Multiply.} \\
= \frac{1}{2} + \frac{1}{6 \cdot 3} & \quad \text{Add.} \\
= \frac{1}{2} + \frac{1}{18} & \\
= \frac{1 \cdot 9 + 1}{2 \cdot 9 + 18} & \\
= \frac{9 + 1}{18 + 18} & \\
= \frac{10}{18} & = \frac{5}{9} \\
= \frac{1}{9} & \\
\end{align*}
\]
§ 3.6

Fractions and Problem Solving
Section Objectives

- Solve Problems by Performing Operations on Fractions or Mixed Numbers
Solve Problems by Performing Operations on Fractions or Mixed Numbers

EXAMPLE

A nacho recipe calls for $\frac{1}{3}$ cup cheddar cheese and $\frac{1}{2}$ cup jalapeño cheese.

Find the total amount of cheese in the recipe.

SOLUTION

$$\frac{1}{3} + \frac{1}{2}$$

$$= \frac{1 \cdot 2}{3 \cdot 2} + \frac{1 \cdot 3}{2 \cdot 3}$$

$$= \frac{2}{6} + \frac{3}{6}$$

$$= \frac{2 + 3}{6} = \frac{5}{6}$$
cups