Chapter 4

Graphing Linear Equations
Chapter Sections

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The Cartesian Coordinate System and Linear Equations in Two Variables
Definitions

A **graph** shows the relationship between two variables in an equation.

The **Cartesian (rectangular) coordinate system** is a grid system used to draw graphs. It is named after its developer, René Descartes (1596-1650).
The two intersecting axis form four quadrants, numbered I through IV.

The horizontal axis is called the x-axis.

The vertical axis is called the y-axis.
Definitions

The point of intersection of the two axes is called the **origin**.

The **coordinates**, or the value of the $x$ and the value of the $y$ determines the point. This is also called an **ordered pair**.
Plotting Points

Plot the point (3, –4). The x-coordinate is 3 and the y-coordinate is –4.
Plot the point \((3, -4)\).
The \(x\)-coordinate is 3 and the \(y\)-coordinate is \(-4\).
Plot the point (3, -4).
The $x$-coordinate is 3 and the $y$-coordinate is -4.
Linear Equations

A **linear equation in two variables** is an equation that can be put in the form

\[ ax + by = c \]

where \( a, b, \) and \( c \) are real numbers.

This is called the **standard form** of an equation.

**Examples:**

\[ 4x - 3y = 12 \]
\[ x + 2y = -35 \]
Solutions to Equations

The **solution** to an equation is the ordered pair that can be substituted into the equation without changing the “validity” of the equation.

Is \((3, 0)\) a solution to the equation \(4x - 3y = 12\)?

\[
4x - 3y = 12 \\
4(3) - 3(0) = 12 \\
12 - 0 = 12 \\
12 = 12
\]  
Yes, it is a solution.
A graph of an equation is an illustration of a set of points whose coordinates satisfy the equation.

A set of points that are in a straight line are collinear.

The points $(-1, 4)$, $(1, 1)$ and $(4, -3)$ are collinear.
4.2

Graphing Linear Equations
Graph by Plotting Points

1. Solve the linear equation for the variable $y$.
2. Select a value for the variable $x$. Substitute this value in the equation for $x$ and find the corresponding value of $y$. Record the ordered pair $(x,y)$.
3. Repeat step 2 with two different values of $x$. This will give you two additional ordered pairs.
4. Plot the three ordered pairs.
5. Draw a straight line through the points.
Graph by Plotting Points

Graph the equation $y = -x + 3$.

Let $x = 2 \quad \Rightarrow \quad y = -2 + 3 \quad \Rightarrow \quad y = 1$
This gives us the point $(2, 1)$.

Let $x = -2 \quad \Rightarrow \quad y = -(-2) + 3 \quad \Rightarrow \quad y = 5$
This gives us the point $(-2, 5)$.

Let $x = 1 \quad \Rightarrow \quad y = -1 + 3 \quad \Rightarrow \quad y = 2$
This gives us the point $(1, 2)$.

Plot the points and draw the line.
Plot the points (2, 1), (-2, 5), and (1, 2).

Draw the line.
Graph Using Intercepts

1. Find the $y$-intercept by setting $x$ in the equation equal to 0 and finding the corresponding value of $y$.
2. Find the $x$-intercept by setting the $y$ in the equation equal to 0 and finding the corresponding value of $x$.
3. Determine a check point by selecting a nonzero value for $x$ and finding the corresponding $y$.
4. Plot the two intercepts and the check point.
5. Draw a straight line through the points.
Graph the equation \(-3y - 2x = -6\).

Let \(x = 0\):

\[-3y - 2(0) = -6 \quad \Rightarrow \quad y = 2\]

This gives us the y-intercept \((0, 2)\).

Let \(y = 0\):

\[-3(0) - 2x = -6 \quad \Rightarrow \quad x = 3\]

This gives us the x-intercept \((3, 0)\).

Let \(x = 2\):

\[-3y - 2(2) = -6 \quad \Rightarrow \quad -3y = -2 \quad \Rightarrow \quad y = \frac{2}{3}\]

This gives us the point \((2, \frac{2}{3})\).

Plot the points and draw the line.
Plot the points $(0, 2)$, $(3, 0)$, and $(2, \frac{2}{3})$.

Draw the line.
4.3

Slope of a Line
The **slope of a line** is the ratio of the vertical change between any two selected points on the line.

\[
\text{slope} = m = \frac{\text{change in } y \text{ (vertical change)}}{\text{change in } x \text{ (horizontal change)}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Consider the points (3, 6) and (1,2).
(3, 6) and (1,2) \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{3 - 1} = \frac{4}{2} = 2 \]

This means the graph is moving up 4 and to the right 2.
The equation for slope is given by:

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{3 - 1} = \frac{4}{2} \]

Simplifying, \( \frac{4}{2} = \frac{2}{1} \), so \( m = 2 \).
Positive & Negative Slopes

Positive Slope
Line rises from left to right

Negative Slope
Line falls from left to right
Horizontal Lines

Every horizontal line has a slope of 0.

$x = 2$
Vertical Lines

The slope of any vertical line is undefined.

$y = -4$
Parallel Lines

Two non-vertical lines with the same slope and different $y$-intercepts are parallel. Any two vertical lines are parallel to each other.

$m_1 = m_2$
Two lines whose slopes are negative reciprocals of each other are **perpendicular** lines. Any vertical line is perpendicular to any horizontal line.

\[ m_1 = \frac{-1}{m_2} \]
Slope-Intercept and Point-Slope Forms
In the **slope-intercept form**, the graph of a linear equation will always be a straight line in the form $y = mx + b$ were $m$ is the slope of the line and $b$ is the y-intercept $(0, b)$.

**Examples:**

- $y = 3x - 4$
  - slope is 3
  - y-intercept is $(0, -4)$

- $y = \text{hand}x + \text{foot}$
  - slope is $\text{hand}$
  - y-intercept is $(0, \text{foot})$
Write the equation \(6x = 8y + 7\) in slope-intercept form.

Solve for \(y\).

\[
6x = 8y + 7 \\
-8y = -6x + 7 \\
y = \frac{-6}{8}x + \frac{7}{8}
\]

Slope is \(\frac{-6}{8}\), \(y\)-intercept is \((0, \frac{7}{8})\).
Point-Slope Form

When the slope and a point on the line are known, we can use the point-slope form to determine the line.

\[ y - y_1 = m(x - x_1) \]

where \( m \) is the slope of the line and \((x_1, y_1)\) is a point.

Example:

point \((2, 3)\) and slope = 
\[ 4: y - 3 = 4(x - 2) \]
\[ y - 3 = 4x - 8 \]
\[ y = 4x - 5 \]
4.5

Graphing Linear Inequalities
A linear inequality results when the equal sign in a linear equation is replaced with an inequality sign (\(<\), \(>\), \(\le\), \(\ge\)).

Examples:

\[ 2y + 3x < 25 \]
\[ -x \geq 15 \]
\[ 4x - y \geq -24 \]
Graph Linear Inequalities

1. Replace the inequality symbol with an equal sign.
2. Draw the graph of the equation in step 1. If the original inequality contained the symbol $\leq$ or $\geq$, draw the graph using a solid line. If the original inequality contained the symbol $<$ or $>$, draw the graph using a dashed line.
3. Select any point not on the line and determine whether this point is a solution to the original inequality. If it is, shade the region on the side of the line containing this point. If it is not a solution, shade the region on the side of the line not containing the point.
Graph \( y < 2x + 1 \).
Functions
Definitions

A **relation** is any set of ordered pairs (points). A **function** is a set of ordered pairs in which each first component (input) corresponds to exactly one second component (output).

Persons

- Tom
- Bob
- Mark
- Bill

Ages

- 21
- 22
- 25
- 20

**FUNCTION**

Persons

- Tom
- Bob
- Mark
- Bill

Ages

- 21
- 22
- 25
- 20

**NOT A FUNCTION**

(Tom can’t be 21 and 22 at the same time.)
Vertical Line Test

If a vertical line can be drawn through any part of a graph and the vertical line intersects another part of the graph, then each value of \( x \) does not correspond exactly to one value of \( y \) and the graph does not represent a function.

If a vertical line cannot be drawn to intersect the graph at more than one point, each value of \( x \) corresponds to exactly one value of \( y \) and the graph represents a function.
Vertical Line Test

FUNCTION

FUNCTION

NOT A FUNCTION
Evaluating Functions

When a function is represented by an equation, **function notation**, \( f(x) \) is used.

\[
y = 3x - 5 \quad \text{is the same as} \quad f(x) = 3x - 5
\]

When a function is **evaluated**, a value is substituted into the function.

\[
f(x) = 3x - 5
\]

\[
f(2) = 3(2) - 5 = 6 - 5 = 1
\]

\[
f(x+1) = 3(x+1) - 5 = 3x + 3 - 5 = 3x - 2
\]