Chapter 9

Roots and Radicals
§ 9.1

Finding Roots
In this section, we introduce a new category of expressions and functions that contain roots.

For example, the reverse operation of squaring a number is finding the square root of the number.

The symbol \( \sqrt{\text{ }} \) that we use to denote the principal square root is called a radical sign. The number under the radical sign is called the radicand. Together we refer to the radical sign and its radicand as a radical expression.
Square Root

Radical Symbol

Radicand, the part underneath the radical symbol.

Radical is the radicand and the radical symbol together.

Blitzer, Introductory Algebra, 5e – Slide #4  Section 9.1
Radical Expressions

Definition of the Principal Square Root

If $a$ is a nonnegative real number, the nonnegative number $b$ such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the principal square root of $a$. 
EXAMPLE

Evaluate: (a) $\sqrt{-16}$  (b) $\sqrt{144} + 25$  (c) $\sqrt{144} + \sqrt{25}$.

SOLUTION

(a) $\sqrt{-16} = \varnothing$

The principal square root of a negative number, -16, is not a real number.

(b) $\sqrt{144} + 25 = \sqrt{169} = 13$

Simplify the radicand. The principal square root of 169 is 13.

(c) $\sqrt{144} + \sqrt{25} = 12 + 5 = 17$

Take the principal square root of 144, 12, and of 25, 5, and then add to get 17.
Finding $n^{th}$ roots.

**EXAMPLES**

$\sqrt[3]{8} = 2$  
Because $2^3 = 8$

$\sqrt[3]{-8} = -2$  
Because $(-2)^3 = -8$

$\sqrt[4]{81} = 3$  
Because $3^4 = 81$

$\sqrt[4]{-81} = -3$  
The negative of the fourth root of 81.

$\sqrt{-36} = \text{not real}$  
The square root of a negative number is not real.
EXAMPLE

Police use the function \( f(x) = \sqrt{20x} \) to estimate the speed of a car, \( f(x) \), in miles per hour, based on the length, \( x \), in feet, of its skid marks upon sudden braking on a dry asphalt road. Use the function to solve the following problem.

A motorist is involved in an accident. A police officer measures the car’s skid marks to be 45 feet long. Estimate the speed at which the motorist was traveling before braking. If the posted speed limit is 35 miles per hour and the motorist tells the officer she was not speeding, should the officer believe her? Explain.
Finding Roots

CONTINUED

SOLUTION

\[ f(x) = \sqrt{20x} \]
\[ f(x) = \sqrt{20 \cdot 45} \]
\[ f(x) = \sqrt{900} \]
\[ f(x) = 30 \]

Use the given function.
Substitute 45 for \( x \).
Simplify the radicand.
Take the square root.

The model indicates that the motorist was traveling at 30 miles per hour at the time of the sudden braking. Since the posted speed limit was 35 miles per hour, the officer should believe that she was not speeding.
Finding Roots

Simplifying $\sqrt{a^2}$

For any real number $a$,

$$\sqrt{a^2} = |a|.$$  

In words, the principal square root of $a^2$ is the absolute value of $a$.

The principal root is the positive root.
## Definition of the Cube Root of a Number

The **cube root** of a real number \(a\) is written \(\sqrt[3]{a}\).

\[ \sqrt[3]{a} = b \] means that \(b^3 = a\).
## Simplifying $\sqrt[3]{a^3}$

For any real number $a$,

$$\sqrt[3]{a^3} = a.$$ 

In words, the cube root of any expression is that expression cubed.
EXAMPLE

Simplify: \( \sqrt[3]{-125} \).

SOLUTION

Begin by expressing the radicand as an expression that is cubed: 
\(-125 = -5^3 \). Then simplify.

\[ \sqrt[3]{-125} = \sqrt[3]{-5^3} = -5 \]

We can check our answer by cubing -5:

\[ -5^3 = -5^3 = -125 \]

By obtaining the original radicand, we know that our simplification is correct.
EXAMPLE

Find the indicated root, or state that the expression is not a real number:

(a) $\sqrt[5]{-1}$ (b) $\sqrt[8]{-1}$.

SOLUTION

(a) $\sqrt[5]{-1} = -1$ because $\left(\sqrt[5]{-1}\right)^5 = (-1)^5 = -1$. An odd root of a negative real number is always negative.

(b) $\sqrt[8]{-1}$ is not a real number because the index, 8, is even and the radicand, -1, is negative. No real number can be raised to the eighth power to give a negative result such as -1. Real numbers to even powers can only result in nonnegative numbers.
### Simplifying $\sqrt[n]{a^n}$

For any real number $a$,

<table>
<thead>
<tr>
<th>Condition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $n$ is even,</td>
<td>$\sqrt[n]{a^n} =</td>
</tr>
<tr>
<td>If $n$ is odd,</td>
<td>$\sqrt[n]{a^n} = a$</td>
</tr>
</tbody>
</table>

In this section, radicals were defined. In the next section, we will learn how to multiply and divide with radicals.
§ 9.2

Multiplying and Dividing Radicals
If $a$ and $b$ represent nonnegative real numbers, then

$$\sqrt{ab} = \sqrt{a} \sqrt{b} \quad \text{and} \quad \sqrt{a} \sqrt{b} = \sqrt{ab}$$

The square root of a product is the product of the square roots.
EXAMPLE

Simplify \( \sqrt{300} \)

\[
\sqrt{300} = \sqrt{100 \cdot 3} \\
= \sqrt{100} \sqrt{3} \\
= 10 \sqrt{3}
\]

The largest perfect square factor in 300 is 100.

Use the product rule.

Simplify.
Multiply:  
(a) $\sqrt[3]{5} \cdot \sqrt[3]{4}$  
(b) $\sqrt[6]{x-5} \cdot \sqrt[6]{x-5}$

**SOLUTION**

In each problem, the *indices are the same*. Thus, we multiply the radicals by multiplying the radicands.

(a) $\sqrt[3]{5} \cdot \sqrt[3]{4} = \sqrt[3]{5 \cdot 4} = \sqrt[3]{20}$

(b) $\sqrt[6]{x-5} \cdot \sqrt[6]{x-5} = \sqrt[6]{(x-5)(x-5)} = \sqrt[6]{x^2 - 10x + 25}$
Simplifying Square Roots

The square root of a variable to an even power equals the variable raised to one half that power.

\[ \sqrt{x^{2n}} = x^n \]

**EXAMPLES**

\[ \sqrt{x^4} = x^2 \]
\[ \sqrt{x^{10}} = x^5 \]
\[ \sqrt{x^{12}} = x^6 \]
Simplifying Square Roots

Simplify $\sqrt{32x^5}$

$\sqrt{32x^5} = \sqrt{16 \cdot 2 \cdot x^4 \cdot x}$

$= \sqrt{16x^4} \sqrt{2x}$

$= 4x^2 \sqrt{2x}$

The largest perfect square factor in $32x^5$ is $16x^4$.

Use the product rule.

Simplify.

Assume variables that appear in the radicand represent nonnegative numbers only.
## Simplifying Radical Expressions by Factoring

A radical expression whose index is $n$ is simplified when its radicand has no factors that are perfect $n$th powers. To simplify, use the following procedure:

1) Write the radicand as the product of two factors, one of which is the greatest perfect $n$th power.

2) Use the product rule to take the $n$th root of each factor.

3) Find the $n$th root of the perfect $n$th power.
EXAMPLE

Simplify by factoring:  (a) \( \sqrt{28} \)  (b) \( 3\sqrt[3]{-32x^2y^3} \).

SOLUTION

(a) \( \sqrt{28} = \sqrt{4 \cdot 7} \)

\[ = \sqrt{4} \cdot \sqrt{7} \]

\[ = 2\sqrt{7} \]

4 is the greatest perfect square that is a factor of 28.

Take the square root of each factor.

Write \( \sqrt{4} \) as 2.

(b) \( 3\sqrt[3]{-32x^2y^3} = 3\sqrt[3]{-8y^3 \cdot 4x^2} \)

\(-8y^3\) is the greatest perfect cube that is a factor of the radicand.
CONTINUED

\[= \sqrt[3]{-8y^3} \cdot \sqrt[3]{4x^2}\]

Factor into two radicals.

\[= -2y\sqrt[3]{4x^2}\]

Take the cube root of \(-8y^3\).
EXAMPLE

Simplify: \( \sqrt{40x^3} \).

SOLUTION

We write the radicand as the product of the greatest perfect square factor and another factor. Because the index of the radical is 2, variables that have exponents that are divisible by 2 are part of the perfect square factor. We use the greatest exponents that are divisible by 2.

\[
\sqrt{40x^3} = \sqrt{4 \cdot 10 \cdot x^2 \cdot x}
\]

Use the greatest even power of each variable.

\[
= \sqrt{4 \cdot x^2 } \cdot \sqrt{10x}
\]

Group the perfect square factors.

\[
= \sqrt{4x^2} \cdot \sqrt{10x}
\]

Factor into two radicals.
A radical is not in simplest form if there are still roots that can be taken. You must look at the expression under the radical sign to verify that you have taken all roots. Since $10x$ contains no factors that are perfect squares, we have indeed now completed the simplification and have taken all roots.
The Quotient Rule for Square Roots

If $a$ and $b$ represent nonnegative real numbers, then

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad \text{and} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

The square root of a quotient is the quotient of the square roots.
EXAMPLE

Simplify $\sqrt{\frac{64x^2}{25}}$

$\sqrt{\frac{64x^2}{25}} = \frac{\sqrt{64x^2}}{\sqrt{25}}$

$= \frac{8x}{5}$

Use the quotient rule.

Simplify by taking the square root of the numerator and the square root of the denominator.
EXAMPLE

Simplify \( \frac{\sqrt{50x^3}}{\sqrt{2x}} \)

\[
\frac{\sqrt{50x^3}}{\sqrt{2x}} = \sqrt{\frac{50x^3}{2x}} \\
= \sqrt{25x^2} \\
= 5x
\]

Use the quotient rule.

Simplify.

Take the square root.
The Product and Quotient Rules for $n^{th}$ Roots

For all real numbers, where the indicated roots represent real numbers,

$$n\sqrt{a} n\sqrt{b} = n\sqrt{ab} \quad \text{and} \quad \frac{n\sqrt{a}}{n\sqrt{b}} = n\sqrt{n} \sqrt{\frac{a}{b}} , b \neq 0$$
EXAMPLE

Simplify $\sqrt[3]{16}$

$\sqrt[3]{40} = \sqrt[3]{8 \cdot 5}$

$= \sqrt[3]{8} \cdot \sqrt[3]{5}$

$= 2\sqrt[3]{5}$

The largest perfect cube factor in 40 is 8.

Use the product rule.

Simplify. The cube root of 8 is 2 because $2^3 = 8$. 

Blitzer, Introductory Algebra, 5e – Slide #31 Section 9.2
EXAMPLE

Simplify: $\sqrt[4]{96x^{11}}$.

SOLUTION

We write the radicand as the product of the greatest 4th power and another factor. Because the index is 4, variables that have exponents that are divisible by 4 are part of the perfect 4th factor. We use the greatest exponents that are divisible by 4.

\[
\sqrt[4]{96x^{11}} = \sqrt[4]{16 \cdot 6 \cdot x^8 \cdot x^3}
\]

Identify perfect 4th factors.

\[
= \sqrt[4]{16x^8 \cdot 6x^3}
\]

Group the perfect 4th factors.

\[
= \sqrt[4]{16x^8} \cdot \sqrt[4]{6x^3}
\]

Factor into two radicals.

\[
= 2x^2 \cdot \sqrt[4]{6x^3}
\]

Simplify the first radical.
EXAMPLE

Multiply and simplify:

(a) $4\sqrt{4x^2 y^3 z^3} \cdot 4\sqrt{8x^4 yz^6}$

(b) $\sqrt[5]{8x^4 y^3 z^3} \cdot \sqrt[5]{8xy^9 z^8}$

SOLUTION

(a) $\sqrt[4]{4x^2 y^3 z^3} \cdot \sqrt[4]{8x^4 yz^6}$

$= \sqrt[4]{4x^2 y^3 z^3 \cdot 8x^4 yz^6}$

$= \sqrt[4]{32x^6 y^4 z^9}$

$= \sqrt[4]{16 \cdot 2x^4 x^2 y^4 z^8 z}$

$= \sqrt[4]{6x^4 y^4 z^8 \cdot x^2 z}$

$= \sqrt[4]{16x^4 y^4 z^8} \cdot \sqrt[4]{2x^2 z}$

Use the product rule.

Multiply.

Identify perfect 4th factors.

Group the perfect 4th factors.

Factor into two radicals.
CONTINUED

(b) \[ \sqrt[5]{8x^4y^3z^3 \cdot 8xy^9z^8} \]

\[ = \sqrt[5]{8x^4y^3z^3 \cdot 8xy^9z^8} \]

\[ = \sqrt[5]{64x^5y^{12}z^{11}} \]

\[ = \sqrt[5]{32 \cdot 2x^5y^{10}y^2z^{10}z} \]

\[ = \sqrt[5]{2x^5y^{10}z^{10}y^2z} \]

\[ = \sqrt[5]{32x^5y^{10}z^{10} \cdot \sqrt[5]{2y^2z}} \]

\[ = 2xy^2z^2 \sqrt[5]{2y^2z} \]

= 2xy^2z^2 \cdot \sqrt[5]{2y^2z}
Important to Remember:

A radical expression of index $n$ is not simplified if you can take any roots, that is, if there are any factors of the radicand that are perfect $n$th powers.

Take all roots.
§ 9.3

Operations with Radicals
Apples to apples, oranges to oranges, ... you can only add “like” things....

Two or more radical expressions that have the same indices and the same radicands are called **like radicals**.

**Like radicals** can be combined under addition in exactly the same way that we combined like terms under addition. Examples of this process follow.

\[ 2 \text{ elephants} + 3 \text{ elephants} = 5 \text{ elephants} \]

but

\[ 5 \text{ tigers} + 3 \text{ gorillas} = ??? \]
Adding and Subtracting Like Radicals

Two or more square roots can be combined using the distributive property provided that they have the same radicand. Such radicals are called like radicals.
Adding Radicals

**EXAMPLE**

Add: \(5\sqrt{13} + 2\sqrt{13}\)

\[
5\sqrt{13} + 2\sqrt{13} = (5 + 2)\sqrt{13}
\]

\[
= 7\sqrt{13}
\]

Apply the distributive property.

Simplify.
EXAMPLE

Subtract: \(5\sqrt{18x} - 2\sqrt{50x}\)

\[
5\sqrt{18x} - 2\sqrt{50x} \\
= 5\sqrt{9 \cdot 2x} - 2\sqrt{25 \cdot 2x} \\
= 5\sqrt{9} \cdot \sqrt{2x} - 2\sqrt{25} \cdot \sqrt{2x} \\
= 5 \cdot 3\sqrt{2x} - 2 \cdot 5\sqrt{2x} \\
= 15\sqrt{2x} - 10\sqrt{2x} \\
= 5\sqrt{2x}
\]

Split radicands into two factors so that one is the largest perfect square.

Take the square root of 9 and 25.

Multiply: 5(3) and 2(5).

We have \textit{like} radicals here and we can subtract them. Simplify.
Combining Radicals

**EXAMPLE**

Simplify (add or subtract) by combining like radical terms:

(a) \(9\sqrt[3]{7} - 4\sqrt[3]{7}\)

(b) \(6\sqrt[3]{7} - \frac{3}{x} \cdot 2\sqrt[3]{7} + 5\frac{3}{x}\).

**SOLUTION**

(a) \(9\sqrt[3]{7} - 4\sqrt[3]{7}\)

\[= 9\cdot\sqrt[3]{7} - 4\cdot\sqrt[3]{7}\]

\[= 5\cdot\sqrt[3]{7}\]

(b) \(6\sqrt[3]{7} - \frac{3}{x} \cdot 2\sqrt[3]{7} + 5\cdot\frac{3}{x}\)

\[= 6\cdot\sqrt[3]{7} + 2\cdot\sqrt[3]{7} - \frac{3}{x} \cdot 2\cdot\sqrt[3]{7} + 5\cdot\frac{3}{x}\]

\[= 6\cdot\sqrt[3]{7} + 2\cdot\sqrt[3]{7} - \frac{3}{x} \cdot 2\cdot\sqrt[3]{7} + 5\cdot\frac{3}{x}\]

\[= 8\cdot\sqrt[3]{7} + 4\cdot\frac{3}{x}\]

In (a) note that these are like radicals, so we can combine them... So let's see... 9 of something less 4 of the same thing is 5 of that thing.

Apply the distributive property.

Simplify.

Apply the distributive property.

Group like terms.

Simplify.
Combining Radicals

Important to remember:

Like radicals have the same indices and radicands. Like radicals can be added or subtracted using the distributive property.

In some cases, you cannot see that radicals are “like” until you simplify them. When attempting to combine radicals, you should simplify the radicals first. Then you may see that you have like radicals that can be combined.

$$\sqrt{12} + \sqrt{75}$$
Are we like? You don’t look like me.

$$2\sqrt{3} + 5\sqrt{3}$$
Yep. I’m 2 square roots of 3 and you are 5 square roots of 3.

$$7\sqrt{3}$$
We have the same indices and radicands. We’re like!
Let’s see...2 of them + 5 of them = 7 of them
Radicals with more than one term are *multiplied* in the same way that polynomials with more than one term are multiplied.

We will now consider multiplication with terms containing square roots.
EXAMPLE

Multiply: \((7 + \sqrt{2})(3 - 5\sqrt{2})\)

\[
(7 + \sqrt{2})(3 - 5\sqrt{2})
= 3 \cdot 7 - 7 \cdot 5\sqrt{2} + 3\sqrt{2} - \sqrt{2} \cdot 5\sqrt{2}
= 21 - 35\sqrt{2} + 3\sqrt{2} - 5\sqrt{4}
= 21 - 35\sqrt{2} + 3\sqrt{2} - 5 \cdot 2
= 21 - 35\sqrt{2} + 3\sqrt{2} - 10
= (21 - 10) + (-35\sqrt{2} + 3\sqrt{2})
= 11 - 32\sqrt{2}
\]

F.O.I.L.
Multiply.
Take the square root of 4.
Multiply 5(2).
Group like terms.
Combine like terms.
EXAMPLE

\[ (\sqrt{3} - \sqrt{2}) (\sqrt{10} - \sqrt{11}) \]

SOLUTION

\[ (\sqrt{3} - \sqrt{2}) (\sqrt{10} - \sqrt{11}) \]

\[ = \sqrt{3} \cdot \sqrt{10} + \sqrt{3} \cdot \sqrt{11} + \sqrt{2} \cdot \sqrt{10} + \sqrt{2} \cdot \sqrt{11} \]

Use FOIL.

\[ = \sqrt{30} - \sqrt{33} - \sqrt{20} + \sqrt{22} \]

Multiply the radicals.
CONTINUED

\[ \sqrt{30} - \sqrt{33} - \sqrt{4 \cdot 5} + \sqrt{22} \]

Factor the third radicand using the greatest perfect square factor.

\[ = \sqrt{30} - \sqrt{33} - \sqrt{4 \cdot 5} + \sqrt{22} \]

Factor the third radicand into two radicals.

\[ = \sqrt{30} - \sqrt{33} - 2\sqrt{5} + \sqrt{22} \]

Simplify.

Note that after simplifying we see that we don’t have any pairs of like radicals. Then we can’t combine these radicals under addition or subtraction and we are done here.
Multiply:

\[(7 + \sqrt{2})(7 - \sqrt{2})\]

\[(7 + \sqrt{2})(7 - \sqrt{2})\]
\[= 7 \cdot 7 - 7 \cdot \sqrt{2} + 7\sqrt{2} - \sqrt{2} \cdot \sqrt{2}\]
\[= 49 - 7\sqrt{2} + 7\sqrt{2} - \sqrt{4}\]
\[= 49 + 0\sqrt{2} - 2\]
\[= 47\]

Note that here we got a \textbf{real number} for our answer. That’s not an accident. That happened because the two expressions we multiplied are special. They are \textit{conjugates} of each other. We will now define “conjugates”.

Blitzer, \textit{Introductory Algebra}, 5e – Slide #47  Section 9.3
Conjugates

Radical expressions that involve the sum and difference of the same two terms are called **conjugates**. To *multiply conjugates*, use

\[(A + B)(A - B) = A^2 - B^2\]

The following three pairs of expressions are conjugates:

\[3 - \sqrt{5} \quad \text{and} \quad 3 + \sqrt{5}\]
\[2 + \sqrt{7} \quad \text{and} \quad 2 - \sqrt{7}\]
\[5 - 2\sqrt{3} \quad \text{and} \quad 5 + 2\sqrt{3}\]

It’s important to remember that **whenever you multiply two conjugates, your result will be a real number**. We will use this fact in the next section.

Now, consider one more example in which we multiply conjugates.
Multiply:

\[(3 + 5\sqrt{2})(3 - 5\sqrt{2})\]

Note that we are multiplying **conjugates** here.

\[
(3 + 5\sqrt{2})(3 - 5\sqrt{2})
= 3 \cdot 3 - 3 \cdot 5\sqrt{2} + 3 \cdot 5\sqrt{2} - 5\sqrt{2} \cdot 5\sqrt{2}
= 9 - 15\sqrt{2} + 15\sqrt{2} - 25\sqrt{4}
= 9 + 0\sqrt{2} - 5 \cdot 2
= 9 - 10
= -1
\]
§ 9.4

Rationalizing the Denominator
Rationalizing the Denominator

It is sometimes easier to work with radical expressions if the denominators do not contain any radicals.

The process of rewriting a radical expression as an equivalent rational expression with no radicals in the denominator is called rationalizing the denominator.
Rationalizing the Denominator

EXAMPLE

Rationalize the Denominator: \( \frac{2}{\sqrt{5}} \)

\[
\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}
\]

Any number divided by itself is 1. Multiplication by 1 does not change the value of the expression.

The denominator contains no radicals, we have rationalized the denominator.
Rationalize the Denominator

**EXAMPLE**

Rationalize the Denominator: \[ \frac{\sqrt{5x}}{\sqrt{18}} \]

\[
\frac{\sqrt{5x}}{\sqrt{18}} = \frac{\sqrt{5x}}{\sqrt{9 \cdot 2}} = \frac{\sqrt{5x}}{3\sqrt{2}} = \frac{\sqrt{5x}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10x}}{6}
\]

Notice what we multiplied by to get rid of the square root of 2 in the denominator.

*When we multiply a square root by itself we get an answer containing no radicals.* For monomial denominators containing square roots, your choice for multiplication can just be that square root over itself. But you should simplify first as we did here – or your problem may be messier than it needs to be.
EXAMPLES

A. \[ \frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{5} \]

B. \[ \frac{3}{\sqrt{7}} = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7} \]

C. \[ \frac{3}{\sqrt{8}} = \frac{3}{\sqrt{4 \cdot 2}} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4} \]
• If the denominator has two terms with one or more square roots, we can rationalize the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

• $A + B$ and $A - B$ are **conjugates**. The product

\[(A + B)(A - B) = A^2 - B^2\]
EXAMPLE

Rationalize the denominator:

\[
\frac{5}{3 + \sqrt{7}}
\]

Multiply numerator and denominator by the conjugate.

\[
= \frac{5}{3 + \sqrt{7}} \left( \frac{3 - \sqrt{7}}{3 - \sqrt{7}} \right)
\]

\[
= \frac{5(3 - \sqrt{7})}{(3 + \sqrt{7})(3 - \sqrt{7})}
\]

\[
= \frac{15 - 5\sqrt{7}}{3^2 - (\sqrt{7})^2}
\]

\[
= \frac{15 - 5\sqrt{7}}{9 - 7}
\]

\[
= \frac{15 - 5\sqrt{7}}{2}
\]

Simplify.
EXAMPLE

Rationalize each denominator: \[ \frac{12}{\sqrt{7} + \sqrt{3}} \]

SOLUTION

The conjugate of the denominator is \( \sqrt{7} - \sqrt{3} \). If we multiply the numerator and the denominator by \( \sqrt{7} - \sqrt{3} \), the simplified denominator will not contain a radical. Therefore, we multiply by 1, choosing \( \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}} \) for 1.

\[
\frac{12}{\sqrt{7} + \sqrt{3}} = \frac{12}{\sqrt{7} + \sqrt{3}} \cdot \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}} \quad \text{Multiply by 1.}
\]
CONTINUED

\[
\frac{12}{\sqrt{7} + \sqrt{3}} = \frac{12}{\sqrt{7} + \sqrt{3}} \cdot \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}}
\]

\[
= \frac{12(\sqrt{7} - \sqrt{3})}{7 - 3}
\]

\[
= \frac{12(\sqrt{7} - \sqrt{3})}{4}
\]

\[
= \frac{3(\sqrt{7} - \sqrt{3})}{A}
\]

Multiply by 1.

\[
A + B (A - B) = A^2 - B^2
\]

Evaluate the exponents.

Subtract.

Divide the numerator and denominator by 4.
Rationalizing Denominators

CONTINUED

\[ 3\left(\sqrt{7} - \sqrt{3}\right) \text{ or } 3\sqrt{7} - 3\sqrt{3} \quad \text{Simplify.} \]

On **Rationalizing the Denominator**...

If the denominator is a *single radical term* with \(n\)th root:
See what expression you would need to multiply by to obtain a perfect \(n\)th power in the denominator. Multiply numerator and denominator by that expression.

If the denominator *contains two terms involving square roots*:
Rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator.

More than two terms in the denominator and rationalizing can get very complicated. Note that you don’t have rules here for those situations. To rationalize simply means to “get the radical out”. By common agreement, we usually rationalize the denominator in a rational expression. We make the denominator “nice” sometimes at the expense of making the numerator messy, but for comparison and other purposes that you will understand later – this choice is best.
In Summary...

**Important to Remember:**

Radical expressions that involve the sum and difference of the same two terms are called **conjugates**. To *multiply conjugates*, use

\[(A + B)(A - B) = A^2 - B^2\]

The process of rewriting a radical expression as an equivalent expression without any radicals in the denominator is called **rationalizing the denominator**.

**GET THOSE RADICALS OUT OF THE DENOMINATOR!!!!**
§ 9.5

Radical Equations
A **radical equation** is an equation in which the variable occurs in a square root, cube root, or any higher root. In this section, you will learn how to solve radical equations.

When the variable occurs in a square root, it is necessary to square both sides of the equation. When you square both sides of an equation, sometimes extra answers creep in, called extraneous roots. For example, consider the following very simple original equation.

\[
x = 2
\]

\[
x^2 = 4
\]

\[
x = \pm 2
\]

Square both sides.
Solve the equation. But -2 does not work in the original equation. Then -2 is an extraneous root. It’s like a hitchhiker that we picked up when we squared both sides. It works only in the squared form and is not a root of the original.
Just to note then....

When you solve a rational equation and must raise both sides to an even power, remember to check your roots. Throw out any extra (extraneous) roots from your solution set.

Another thing that you should know is ... if your equation contains two or more square root expressions, you will need to isolate one square root expression, square both sides, and then you may have to repeat the process.

You may have to square both sides twice.
### Solving Radical Equations Containing \( n \)th Roots

1. If necessary, arrange terms so that one radical is isolated on one side of the equation.

2. Raise both sides of the equation to the \( n \)th power to eliminate the \( n \)th root.

3. Solve the resulting equation. If this equation still contains radicals, repeat steps 1 and 2.

4. Check all proposed solutions in the original equation.
Solving Radical Equations Containing Square Roots

1. If necessary, arrange the terms so that the one radical is isolated on one side of the equation.
2. Square both sides of the equation to eliminate the square root.
3. Solve the resulting equation.
4. Check all proposed solutions in the original equation.
EXAMPLE

Solve: \( 7 = 5 + \sqrt{x} \)

Step 1: Isolate the radical.

\[
7 = 5 + \sqrt{x} \\
2 = \sqrt{x}
\]

Subtract 5 from both sides.

Step 2: Square both sides.

\[
2^2 = (\sqrt{x})^2 \\
4 = x
\]

Step 3: Solve the resulting equation.

\( x = 4 \)
Solve: \[ 7 = 5 + \sqrt{x} \]

**Step 4:** Check the proposed solution in the original equation. Let \( x = 4 \).

\[
\begin{align*}
7 & \neq 5 + \sqrt{4} \\
7 & \neq 5 + 2 \\
7 & = 7 \quad \text{true}
\end{align*}
\]
EXAMPLE

Solve: \( x = 6 + \sqrt{2x + 3} \)

Step 1: Isolate the radical.
\[
x = 6 + \sqrt{2x + 3}
\]
\[
x - 6 = \sqrt{2x + 3}
\]
Subtract 6 from both sides.

Step 2: Square both sides.
\[
(x - 6)^2 = (\sqrt{2x + 3})^2
\]
\[
x^2 - 12x + 36 = 2x + 3
\]
Simplify.
Use \((A-B)^2 = A^2 - 2AB + B^2\)
Solve: \( x = 6 + \sqrt{2x + 3} \)

**Step 3: Solve the resulting equation.**

\[
\begin{align*}
    x^2 - 12x + 36 &= 2x + 3 \\
    x^2 - 14x + 33 &= 0 \\
    (x - 11)(x - 3) &= 0 \\
    x - 11 &= 0 \quad \text{or} \quad x - 3 = 0 \\
    x &= 11 \quad \text{or} \quad x = 3
\end{align*}
\]

The resulting equation is quadratic. Write in standard form, subtract 2x and subtract 3 from both sides.

Factor and solve.
Solve: \( x = 6 + \sqrt{2x + 3} \)

**Step 4: Check the proposed solutions in the original equation.**

<table>
<thead>
<tr>
<th>( x = 11 )</th>
<th>( x = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 11 \neq 6 + \sqrt{2(11) + 3} )</td>
<td>( 3 = 6 + \sqrt{2(3) + 3} )</td>
</tr>
<tr>
<td>( 11 \neq 6 + \sqrt{25} )</td>
<td>( 3 \neq 6 + \sqrt{9} )</td>
</tr>
<tr>
<td>( 11 \neq 6 + 5 )</td>
<td>( 3 \neq 6 + 3 )</td>
</tr>
<tr>
<td>( 11 = 11 ) \ true</td>
<td>( 3 \neq 9 ) \ false</td>
</tr>
</tbody>
</table>

Thus, 3 is an extraneous solution. The only solution is 11.
EXAMPLE

Solve: $\sqrt{2x+5} + 11 = 6$.

SOLUTION

1) Isolate a radical on one side. The radical, $\sqrt{2x+5}$, can be isolated by subtracting 11 from both sides. We obtain

$$\sqrt{2x+5} = -5.$$

2) Raise both sides to the $n$th power. Because $n$, the index, is 2, we square both sides.

$$\sqrt[2]{2x+5}^2 = (-5)^2$$

$$2x + 5 = 25$$

Simplify.
Solving Radical Equations

CONTINUED

3) Solve the resulting equation.

\[ 2x + 5 = 25 \]
\[ 2x = 20 \]
\[ x = 10 \]

This is the equation from step 2.
Subtract 5 from both sides.
Divide both sides by 2.

4) Check the proposed solution in the original equation.

Check 10:

\[ \sqrt{2x + 5} + 11 = 6 \]
\[ \sqrt{2(10) + 5} + 11 \neq 6 \]
\[ \sqrt{25} + 11 \neq 6 \]

\[ 5 + 11 \neq 6 \]
\[ 16 = 6 \text{ false} \]

Therefore there is no solution to the equation.
EXAMPLE

Solve: \(3x - \sqrt{3x + 7} = -5\).

SOLUTION

1) Isolate a radical on one side. The radical, \(\sqrt{3x + 7}\), can be isolated by subtracting \(3x\) from both sides. We obtain

\[-\sqrt{3x + 7} = -5 - 3x.\]

2) Raise both sides to the \(n\)th power. Because \(n\), the index, is 2, we square both sides.

\[(-\sqrt{3x + 7})^2 = (-5 - 3x)^2\]

\[3x + 7 = 25 + 30x + 9x^2\]

Simplify. Use the special formula \(A + B)^2 = A^2 + 2AB + B^2\).
CONTINUED

3) Solve the resulting equation. Because of the $x^2$-term, the resulting equation is a quadratic equation. We need to write this quadratic equation in standard form. We can obtain zero on the left side by subtracting $3x$ and 7 from both sides.

$$3x + 7 = 9x^2 + 30x + 25$$
Equation from step 2.

$$0 = 9x^2 + 27x + 18$$
Subtract $3x$ and 7 from both sides.

$$0 = 9(x^2 + 3x + 2)$$
Factor out the GCF, 9.

$$0 = x^2 + 3x + 2$$
Divide both sides by 9.

$$0 = (x + 2)(x + 1)$$
Factor the right side.
Solving Radical Equations

CONTINUED

\[ x + 2 = 0 \quad x + 1 = 0 \]
\[ x = -2 \quad x = -1 \]

Set each factor equal to 0.

Solve for \( x \).

4) Check the proposed solutions in the original equation.

Check -2:
\[ 3x - \sqrt{3x+7} = -5 \]
\[ 3(-2) - \sqrt{3(-2)+7} = -5 \]
\[ -6 - \sqrt{1} \neq -5 \]
\[ -6 - 1 \neq -5 \]
\[ -7 = -5 \text{ false} \]

Check -1:
\[ 3x - \sqrt{3x+7} = -5 \]
\[ 3(-1) - \sqrt{3(-1)+7} = -5 \]
\[ -3 - \sqrt{4} \neq -5 \]
\[ -3 - 2 \neq -5 \]
\[ -5 = -5 \text{ true} \]

The solution is -1. The solution set is \{ -1 \}. 

Blitzer, Introductory Algebra, 5e – Slide #75 Section 9.5
Solving Radical Equations

**EXAMPLE**

Solve: $\sqrt{x-4} + \sqrt{x+4} = 4$.

**SOLUTION**

1) **Isolate a radical on one side.** The radical, $\sqrt{x-4}$, can be isolated by subtracting $\sqrt{x+4}$ from both sides. We obtain

$$\sqrt{x-4} = 4 - \sqrt{x+4}.$$ 

2) **Raise both sides to the $n$th power.** Because $n$, the index, is 2, we square both sides.

$$\left(\sqrt{x-4}\right)^2 = \left(4 - \sqrt{x+4}\right)^2$$

$$x - 4 = 16 - 8\sqrt{x+4} + x + 4$$


Blitzer, *Introductory Algebra*, 5e – Slide #76  Section 9.5
CONTINUED

\[ x - 4 = 20 + x - 8\sqrt{x + 4} \]

Combine like terms.

1) Isolate a radical on one side. The radical, \( \sqrt{x + 4} \), can be isolated by subtracting 20 + \( x \) from both sides and then dividing both sides by -8. We obtain

\[ 3 = \sqrt{x + 4}. \]

2) Raise both sides to the \( n \)th power. Because \( n \), the index, is 2, we square both sides.

\[ 3^2 = (\sqrt{x + 4})^2 \]

\[ 9 = x + 4 \]

Square the 3 and the \( \sqrt{x + 4} \).
CONTINUED

3) Solve the resulting equation.

\[ 9 = x + 4 \]

This is the equation from the last step.

\[ 5 = x \]

Subtract 4 from both sides.

3) Check the proposed solution in the original equation.

Check 5:

\[ \sqrt{x - 4} + \sqrt{x + 4} = 4 \]

\[ \sqrt{1} + \sqrt{9} \neq 4 \]

\[ 1 + 3 \neq 4 \]

The solution is 5. The solution set is \{5\}. 

\[ 4 = 4 \quad \text{true} \]
In Summary…

Solving Radical Equations Containing nth Roots

1. Isolate one radical on one side of the equation.
2. Raise both sides to the $n$th power.
3. Solve the resulting equation.
4. **Check** proposed solutions in the *original equation*.

Sometimes proposed solutions will work in the final simplified form of the original equation, but will not work in the original equation itself. These imposter roots that sometimes slip in when we square both sides of an equation are called *extraneous roots*. 
§ 9.6

Rational Exponents
Rational Exponents

Rational exponents have been defined in such a way so as to make their properties the same as the properties for integer exponents.

In this section we explore the meaning of a base raised to a rational (fractional) exponent.

We will also discover how we can use rational exponents to simplify radical expressions.
The Definition of $a^{1/n}$

If $\sqrt[n]{a}$ represents a real number and $n \geq 2$ is an integer, then

$$a^{1/n} = \sqrt[n]{a}.$$ 

If $a$ is negative, $n$ must be odd. If $a$ is nonnegative, $n$ can be any index.
Definition of $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ represents a real number and $n \geq 2$ is an integer, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

The denominator of the rational exponent is the radical’s index.
EXAMPLES

Find $16^2$, $-16^2$ and $(-16)^2$

$16^2 = \sqrt{16} = 4$

$-16^2 = -\sqrt{16} = -4$

$(-16)^2 = \sqrt{-16} = \text{not real}$

The $\frac{1}{2}$ power is the square root.

The base is 16 and the negative sign is not affected by the exponent.

The base is -16 and the negative sign is affected by the exponent. The square root of a negative is not real.
Rational Exponents

**EXAMPLES**

Find $\frac{1}{8^3}$, $-\frac{1}{8^3}$ and $(−8)^{\frac{1}{3}}$

\[
\frac{1}{8^3} = \frac{1}{3\sqrt[3]{8}} = \frac{1}{2} \\
-\frac{1}{8^3} = -\frac{1}{3\sqrt[3]{8}} = -\frac{1}{2} \\
(−8)^{\frac{1}{3}} = \frac{1}{3\sqrt[3]{-8}} = -\frac{1}{2}
\]

The base is 8 and the negative sign is not affected by the exponent.

The 1/3 power is the cube root.

The base is -8 and the negative sign is affected by the exponent. The square root of a negative is not real.
EXAMPLE

Use radical notation to rewrite each expression. Simplify, if possible:

(a) $\sqrt[3]{xy^4} \quad$ (b) $100^{\frac{1}{2}} \quad$ (c) $\sqrt[3]{64^{\frac{1}{3}}}$.

SOLUTION

(a) $\sqrt[3]{xy^4} = \sqrt[3]{3xy^4}$

(b) $100^{\frac{1}{2}} = \sqrt{100} = 10$

(c) $\sqrt[3]{64^{\frac{1}{3}}} = \sqrt[3]{-64} = -4$
EXAMPLE

Rewrite with rational exponents:

(a) $\sqrt[5]{13x}$  (b) $\sqrt{x^5}$.

SOLUTION

Parentheses are needed in part (a) to show that the entire radicand becomes the base.

(a) $\sqrt[5]{13x} = (3x)^{\frac{1}{5}}$

(b) $\sqrt{x^5} = (x^5)^{\frac{1}{2}} = x^{5 \cdot \frac{1}{2}} = x^2$
Rational Exponents

The Definition of $a^{m/n}$

If $\sqrt[n]{a}$ represents a real number, $\frac{m}{n}$ is a positive rational number reduced to lowest terms, and $n \geq 2$ is an integer, then

\[ a^{m/n} = \left(\sqrt[n]{a}\right)^m \]

and

\[ a^{m/n} = \sqrt[n]{a^m}. \]
Rational Exponents

Definition of $a^{\frac{m}{n}}$

If $\sqrt[n]{a}$ represents a real number and $\frac{m}{n}$ is a positive rational number, $n \geq 2$, then

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}^{m}\right) = \sqrt[n]{a^m}$$
EXAMPLE

Write in radical form and simplify: \( \frac{2}{8^{\frac{3}{2}}} \)

\[
8^{\frac{3}{2}} = \sqrt[2]{8^3} = 2^2 = 4
\]
Write in radical form and simplify: \(-8^{\frac{2}{3}}\)

\[-8^{\frac{2}{3}} = -\left(\sqrt[3]{8^2}\right) = -2^2 = -4\]

The base is 8 and the negative sign is not affected by the exponent.
Write in radical form and simplify: \((-8)^{\frac{2}{3}}\)

\[ (-8)^{\frac{2}{3}} = \left( \sqrt[3]{-8} \right)^2 = (-2)^2 = 4 \]

The base is -8. The negative sign is affected by the exponent.
EXAMPLE

Use radical notation to rewrite each expression and simplify:

(a) \( 25^{\frac{3}{2}} \)  
(b) \( -27^{\frac{2}{3}} \).

SOLUTION

(a) \[ 25^{\frac{3}{2}} = \sqrt[3]{25^2} = 5^3 = 125 \]

(b) \[ -27^{\frac{2}{3}} = \sqrt[3]{-27^2} = -3^2 = 9 \]
Rational Exponents

EXAMPLE

Rewrite with rational exponents:

(a) \( \sqrt[4]{x^7} \)  (b) \( \sqrt[3]{11xy^3} \)

SOLUTION

(a) \( \sqrt[4]{x^7} = x^{\frac{7}{4}} \)

(b) \( \sqrt[3]{11xy^3} = \sqrt[3]{11xy^3} \)
Rational Exponents

The Definition of $a^{-m/n}$

If $a^{m/n}$ is a nonzero real number, then

$$a^{-m/n} = \frac{1}{a^{m/n}}.$$
EXAMPLE

The Galapagos Islands, lying 600 miles west of Ecuador, are famed for their extraordinary wildlife. The function

\[ f(x) = 29x^{\frac{1}{3}} \]

models the number of plant species, \( f(x) \), on the various islands of the Galapagos chain in terms of the area, \( x \), in square miles, of a particular island. Use the function to solve the following problem.

How many species of plants are on a Galapagos island that has an area of 27 square miles?
Because we are interested in how many species of plants there are on a Galapagos island having an area of 27 square miles, substitute 27 for $x$. Then calculate $f(x)$.

\[ f(x) = 29 \cdot x^\frac{1}{3} \]

This is the given formula.

\[ f(27) = 29 \cdot 27^\frac{1}{3} \]

Replace $x$ with 27.

\[ f(27) = 29 \cdot \sqrt[3]{27} \]

Rewrite $27^\frac{1}{3}$ as $\sqrt[3]{27}$.

\[ f(27) = 29 \cdot 3 \]

Evaluate the cube root.

\[ f(27) = 87 \]

Multiply.

A Galapagos island having an area of 27 square miles contains approximately 87 plant species.
### Properties of Rational Exponents

If $m$ and $n$ are rational exponents, and $a$ and $b$ are real numbers for which the following expressions are defined, then

1) $b^m \cdot b^n = b^{m+n}$  
   When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

2) $\frac{b^m}{b^n} = b^{m-n}$  
   When dividing exponential expressions with the same base, subtract the exponents. Use this difference as the exponent of the common base.
### Properties of Rational Exponents

If $m$ and $n$ are rational exponents, and $a$ and $b$ are real numbers for which the following expressions are defined, then

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3)</td>
<td>When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.</td>
</tr>
<tr>
<td>$a^{m/n} = b^{mn}$</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>When a <em>product</em> (not sum) is raised to a power, raise each factor to that power and multiply.</td>
</tr>
<tr>
<td>$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$</td>
<td></td>
</tr>
<tr>
<td>5)</td>
<td>When a quotient is raised to a power, raise the numerator to that power and divide by the denominator to that power.</td>
</tr>
<tr>
<td>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE

Simplify: (a) \( \frac{x^{\frac{3}{7}}}{x^{\frac{1}{7}}} \)  
(b) \( \left( x^4 y^{\frac{2}{5}} \right)^{\frac{1}{3}} \)  
(c) \( \frac{5^4 \cdot 5^2}{5^4} \).

SOLUTION

\[
\begin{align*}
\text{(a)} \quad & \frac{x^{\frac{3}{7}}}{x^{\frac{1}{7}}} = x^{\frac{3}{7} - \frac{1}{7}} \\
& = x^{\frac{2}{7}} \\
\end{align*}
\]

To divide with the same base, subtract exponents.

Subtract.
Rational Exponents

CONTINUED

(b) \( \left( \frac{1}{x^4} \cdot \frac{2}{y^5} \right)^{\frac{1}{3}} = \left( \frac{1}{x^4} \right)^{\frac{1}{3}} \left( \frac{2}{y^5} \right)^{\frac{1}{3}} \)

\[ = \frac{1}{x^{12}} \cdot \frac{2}{y^{15}} \]

\[ = \frac{1}{x^{12}} \]

\[ = \frac{2}{y^{15}} \]

To raise a product to a power, raise each factor to the power.

Multiply: \( \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12} \) and \( -\frac{2}{5} \cdot \frac{1}{3} = -\frac{2}{15} \).

Rewrite with positive exponents.

(c) \( \frac{3^4 \cdot 5^2}{1^3} = \frac{3^{4+1}}{1^1} = \frac{3^{4+2}}{5^1} = \frac{5}{1} = 5 \)

\[ = 5^{\frac{1}{4}} = 5^{\frac{4}{4}} = 5^1 = 5 \]
## Simplifying Radical Expressions Using Rational Exponents

1) Rewrite each radical expression as an exponential expression with a rational exponent.

2) Simplify using properties of rational exponents.

3) Rewrite in radical notation if rational exponents still appear.
Rational Exponents

EXAMPLE

Use rational exponents to simplify: (a) \(\sqrt[6]{ab^2} \cdot \sqrt[3]{a^2b}\) (b) \(\sqrt[5]{3\sqrt[3]{2x}}\).

SOLUTION

(a) \(\sqrt[6]{ab^2} \cdot \sqrt[3]{a^2b} = \left(\sqrt[6]{b^2}\right)^{\frac{1}{6}} \cdot \left(\sqrt[3]{b}\right)^{\frac{1}{3}}\)

= \(a^{\frac{1}{6}}b^{\frac{1}{3}}\)

Raise each factor in parentheses to its related power.

To raise powers to powers, multiply.

Reorder the factors.

To multiply with the same base, add exponents.

Rewrite as exponential expressions.
Rational Exponents

CONTINUED

\[
\begin{align*}
&\quad \quad = a^{\frac{5}{6}} b^{\frac{2}{3}} \\
&= a^{\frac{5}{6}} b^{\frac{4}{6}} \\
&= a^{\frac{5}{6}} b^{\frac{4}{6}} \\
&= \sqrt[6]{a^5 b^4} \\
\end{align*}
\]

Add.

Rewrite exponents with common denominators.

Factor 1/6 out of the exponents.

Rewrite in radical notation.

Write the radicand as an exponential expression.

Write the entire expression in exponential form.

To raise powers to powers, multiply the exponents.

Rewrite in radical notation.

(b) \( \sqrt[3]{\sqrt[2]{2x}} = \sqrt[5]{(\sqrt[3]{x})^{\frac{1}{5}}} \)

\[
\begin{align*}
&= \sqrt[5]{\sqrt[3]{x}^{\frac{1}{5}}} \\
&= \sqrt[15]{x^{\frac{1}{5}}} \\
&= \sqrt[15]{2x}
\end{align*}
\]
Rational Exponents

Important to Remember:

Properties of integer exponents also hold for rational exponents.

Remember those exponent rules we learned before? Well, they still hold, even when the exponents are fractions.

An expression with rational exponents is simplified when no parentheses appear, no powers are raised to powers, each base occurs once, and no negative or zero exponents appear.
Rational Exponents

Important to Remember:

Some radical expressions can be simplified using rational exponents. Rewrite the expression using rational exponents, simplify, and rewrite in radical notation if rational exponents still appear.

Didn’t memorize any radical rules? Then rewrite your expression without the radical sign using rational exponents instead. Now you can use your exponent rules to simplify the radical expression.