Chapter 22
Electric Fields

In this chapter we will introduce the concept of an electric field. As long as charges are stationary, Coulomb’s law describes adequately the forces among charges. If the charges are not stationary we must use an alternative approach by introducing the electric field (symbol $\mathbf{E}$). In connection with the electric field, the following topics will be covered:

- Calculating the electric field generated by a point charge.
- Using the principle of superposition to determine the electric field created by a collection of point charges as well as continuous charge distributions.
- Once the electric field at a point $P$ is known, calculating the electric force on any charge placed at $P$.
- Defining the notion of an “electric dipole.” Determining the net force, the net torque, exerted on an electric dipole by a uniform electric field, as well as the dipole potential energy.
In Chapter 21 we discussed Coulomb’s law, which gives the force between two point charges. The law is written in such a way as to imply that $q_2$ acts on $q_1$ at a distance $r$ \textbf{instantaneously} (“action at a distance”):

Electric interactions propagate in empty space with a large but \textbf{finite} speed ($c = 3 \times 10^8$ m/s). In order to take into account correctly the finite speed at which these interactions propagate, we have to abandon the “action at a distance” point of view and still be able to explain how $q_1$ knows about the presence of $q_2$ . The solution is to introduce the new concept of an \textbf{electric field} vector as follows: Point charge $q_1$ does not exert a force directly on $q_2$. Instead, $q_1$ creates in its vicinity an electric field that exerts a force on $q_2$.

\begin{align*}
\text{charge } q_1 & \rightarrow \text{ generates electric field } \vec{E} \rightarrow \vec{E} \text{ exerts a force } \vec{F} \text{ on } q_2 \\
F &= \frac{1}{4\pi\varepsilon_0} \frac{|q_1||q_2|}{r^2}
\end{align*}
Consider the positively charged rod shown in the figure. For every point $P$ in the vicinity of the rod we define the electric field vector $\vec{E}$ as follows:

1. We place a **positive** test charge $q_0$ at point $P$.
2. We measure the electrostatic force $\vec{F}$ exerted on $q_0$ by the charged rod.
3. We define the electric field vector $\vec{E}$ at point $P$ as:

$$\vec{E} = \frac{\vec{F}}{q_0}.$$  

SI Units: $\text{N/C}$

From the definition it follows that $\vec{E}$ is parallel to $\vec{F}$.

**Note:** We assume that the test charge $q_0$ is small enough so that its presence at point $P$ does not affect the charge distribution on the rod and thus alter the electric field vector $\vec{E}$ we are trying to determine.

(22-3)
Electric Field Generated by a Point Charge

Consider the positive charge $q$ shown in the figure. At point $P$ a distance $r$ from $q$ we place the test charge $q_0$. The force exerted on $q_0$ by $q$ is equal to:

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \frac{|q||q_0|}{r^2}$$

The magnitude of $\mathbf{E}$ is a positive number. In terms of direction, $\mathbf{E}$ points radially outward as shown in the figure.

If $q$ were a negative charge the magnitude of $\mathbf{E}$ would remain the same. The direction of $\mathbf{E}$ would point radially inward instead.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}$$

(22-4)
Electric Field Generated by a Group of Point Charges. Superposition

The net electric electric field \( \vec{E} \) generated by a group of point charges is equal to the vector sum of the electric field vectors generated by each charge.

In the example shown in the figure, \( \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \).

Here \( \vec{E}_1, \vec{E}_2, \) and \( \vec{E}_3 \) are the electric field vectors generated by \( q_1, q_1, \) and \( q_3, \) respectively.

Note: \( \vec{E}_1, \vec{E}_2, \) and \( \vec{E}_3 \) must be added as vectors:

\[
E_x = E_{1x} + E_{2x} + E_{3x}, \quad E_y = E_{1y} + E_{2y} + E_{3y}, \quad E_z = E_{1z} + E_{2z} + E_{3z}
\]

(22-5)
Electric Dipole

A system of two equal charges of opposite sign \( \pm q \) placed at a distance \( d \) apart is known as an "electric dipole." For every electric dipole we associate a vector known as "the electric dipole moment" (symbol \( \vec{p} \)) defined as follows:

The magnitude \( p = qd \)

The direction of \( \vec{p} \) is along the line that connects the two charges and points from \(-q\) to \(+q\).

Many molecules have a built-in electric dipole moment. An example is the water molecule (H\(_2\)O).

The bonding between the O atom and the two H atoms involves the sharing of 10 valence electrons (8 from O and 1 from each H atom).

The 10 valence electrons have the tendency to remain closer to the O atom. Thus the O side is more negative than the H side of the H\(_2\)O molecule.  

(22-6)
Electric Field Generated by an Electric Dipole

We will determine the electric field $\vec{E}$ generated by the electric dipole shown in the figure using the principle of superposition. The positive charge generates at $P$ an electric field whose magnitude $E_{(+)} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2_+}$. The negative charge creates an electric field with magnitude $E_{(-)} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2_-}$.

The net electric field at $P$ is $E = E_{(+)} - E_{(-)}$.

$$E = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{r^2_+} - \frac{q}{r^2_-} \right) = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{ \left( \frac{z-d}{2} \right)^2 } - \frac{q}{ \left( \frac{z+d}{2} \right)^2 } \right)$$

We assume: $\frac{d}{2z} \ll 1$

$$E = \frac{q}{4\pi\varepsilon_0 z^2} \left[ \left( 1 - \frac{d}{2z} \right)^{-2} - \left( 1 + \frac{d}{2z} \right)^{-2} \right]$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2} \left[ \left( 1 + \frac{d}{z} \right) - \left( 1 - \frac{d}{z} \right) \right] = \frac{qd}{2\pi\varepsilon_0 z^3} = \frac{1}{2\pi\varepsilon_0} \frac{p}{z^3}$$

$$(22-7)$$
Electric Field Generated by a Continuous Charge Distribution

Consider the continuous charge distribution shown in the figure. We assume that we know the volume density $\rho$ of the electric charge. This is defined as $\rho = \frac{dq}{dV}$ (Units: C/m$^3$).

Our goal is to determine the electric field $d\vec{E}$ generated by the distribution at a given point $P$. This type of problem can be solved using the principle of superposition as described below.

1. Divide the charge distribution into "elements" of volume $dV$. Each element has charge $dq = \rho dV$. We assume that point $P$ is at a distance $r$ from $dq$.

2. Determine the electric field $d\vec{E}$ generated by $dq$ at point $P$.

The magnitude $dE$ of $d\vec{E}$ is given by the equation $dE = \frac{dq}{4\pi \epsilon_0 r^2}$.

3. Sum all the contributions: $\vec{E} = \frac{1}{4\pi \epsilon_0} \int \frac{\rho dV \hat{r}}{r^2}$.

(22-8)
Example: Determine the electric field $\vec{E}$ generated at point $P$ by a uniformly charged ring of radius $R$ and total charge $q$. Point $P$ lies on the normal to the ring plane that passes through the ring center $C$, at a distance $z$. Consider the charge element of length $dS$ and charge $dq$ shown in the figure. The distance between the element and point $P$ is $r = \sqrt{z^2 + R^2}$.

The charge $dq$ generates at $P$ an electric field of magnitude $dE$ that points outward along the line $AP$:

$$dE = \frac{dq}{4\pi \varepsilon_0 r^2}.$$  

The $z$-component of $d\vec{E}$ is given by

$$dE_z = dE \cos \theta.$$  

From triangle PAC we have: $\cos \theta = z / r$

$$\rightarrow dE_z = \frac{z dq}{4\pi \varepsilon_0 r^3} = \frac{z dq}{4\pi \varepsilon_0 \left(z^2 + R^2\right)^{3/2}}.$$  

$$E_z = \int dE_z = \frac{z q}{4\pi \varepsilon_0 \left(z^2 + R^2\right)^{3/2}}$$

\begin{equation}(22-9)\end{equation}
Electric Field Lines. In the 19th century Michael Faraday introduced the concept of electric field lines, which help visualize the electric field vector $\vec{E}$ without using mathematics. For the relation between the electric field lines and $\vec{E}$:

1. At any point $P$ the electric field vector $\vec{E}$ is tangent to the electric field lines.

2. The magnitude of the electric field vector $\vec{E}$ is proportional to the density of the electric field lines.

Mathematically:

$$E_P > E_Q$$

*(22-10)*
Example 2: Electric field lines of an electric field generated by an infinitely large plane uniformly charged. In the next chapter we will see that the electric field generated by such a plane has the form shown in fig. $b$.

1. The electric field on either side of the plane has a constant magnitude.
2. The electric field vector is perpendicular to the charge plane.
3. The electric field vector $\vec{E}$ points away from the plane.

The corresponding electric field lines are given in fig. $c$.

Note: For a negatively charged plane the electric field lines point inward.
3. Electric field lines extend away from **positive charges** (where they originate) and toward **negative charges** (where they terminate).

Example 1: Electric field lines of a negative point charge - $q$:

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{|q|}{r^2}
\]

- The electric field lines point toward the point charge.
- The direction of the lines gives the direction of $\vec{E}$.
- The density of the lines/unit area increases as the distance from $-q$ decreases.

**Note**: In the case of a positive point charge the electric field lines have the same form but they point **outward**.
Example 3:
Electric field lines generated by an electric dipole (a positive and a negative point charge of the same size but of opposite sign)

Example 4:
Electric field lines generated by two equal positive point charges

(22-13)
Forces and Torques Exerted on Electric Dipoles by a Uniform Electric Field

Consider the electric dipole shown in the figure in the presence of a uniform (constant magnitude and direction) electric field $\vec{E}$ along the $x$-axis. The electric field exerts a force $F_+ = qE$ on the positive charge and a force $F_- = -qE$ on the negative charge. The net force on the dipole is $F_{\text{net}} = qE - qE = 0$.

The net torque generated by $F_+$ and $F_-$ about the dipole center is

$$\tau = \tau_+ + \tau_- = -\left|F_+\right| \frac{d}{2} \sin \theta - \left|F_-\right| \frac{d}{2} \sin \theta = -qEd \sin \theta = -pE \sin \theta$$

In vector form: $\vec{\tau} = \vec{p} \times \vec{E}$

The electric dipole in a uniform electric field does not move but can rotate about its center.

$$\vec{F}_{\text{net}} = 0 \quad \vec{\tau} = \vec{p} \times \vec{E} \quad \text{(22-14)}$$
Potential Energy of an Electric Dipole in a Uniform Electric Field

\[ U = -pE \cos \theta \]

\[ U = -\vec{p} \cdot \vec{E} \]

At point \( A (\theta = 0) \), \( U \) has a minimum value \( U_{\text{min}} = -pE \).
It is a position of stable equilibrium.

At point \( B (\theta = 180^\circ) \), \( U \) has a maximum value \( U_{\text{max}} = +pE \).
It is a position of unstable equilibrium.
Consider the electric dipole in fig. *a*. It has an electric dipole moment $\vec{p}$ and is positioned so that $\vec{p}$ is at an angle $\theta_i$ with respect to a uniform electric field $\vec{E}$.

An external agent rotates the electric dipole and brings it to its final position shown in fig. *b*. In this position $\vec{p}$ is at an angle $\theta_f$ with respect to $\vec{E}$.

The work $W$ done by the external agent on the dipole is equal to the difference between the initial and final potential energy of the dipole:

$$W = U_f - U_i = -pE \cos \theta_f - pE \cos \theta_i$$

$$W = pE (\cos \theta_i - \cos \theta_f)$$

(22-16)