2.1 Linear Equations in One Variable

1. Decide whether a number is a solution of a linear equation.
2. Solve linear equations by using the addition and multiplication properties of equality.
3. Solve linear equations by using the distributive property.
4. Solve linear equations with fractions or decimals.
5. Identify conditional equations, contradictions, and identities.
Equations and inequalities compare algebraic expressions.

An equation always contains an equals sign, while an expression does not.

\[3x - 7 = 2\]

- **Left side**
- **Right side**

\[3x - 7\]

Equation (to solve)

Expression (to simplify or evaluate)
A linear equation in one variable can be written in the form

$$Ax + B = C,$$

where $A$, $B$, and $C$ are real numbers, with $A \neq 0$.

A linear equation is a first-degree equation, since the greatest power on the variable is 1.
EXAMPLE

Decide whether each of the following is an equation or expression.

a. $9x = 10$  
   *equation*

b. $9x + 10$  
   *expression*

c. $3 + 5x - 8x + 9$  
   *expression*

d. $3 + 5x = -8x + 9$  
   *equation*
Objective 1

Decide whether a number is a solution of a linear equation.
If the variable in an equation can be replaced by a real number that makes the statement true, then that number is a **solution** of the equation.

An equation is *solved* by finding its **solution set**, the set of all solutions.

**Equivalent equations** are related equations that have the same solution set.
Objective 2

Solve linear equations by using the addition and multiplication properties of equality.
Addition and Multiplication Properties of Equality

Addition Property of Equality
For all real numbers $A$, $B$, and $C$, the equations

$$A = B \quad \text{and} \quad A + C = B + C$$

are equivalent.
That is, the same number may be added to each side of an equation without changing the solution set.

Multiplication Property of Equality
For all real numbers $A$, and $B$, and for $C \neq 0$, the equations

$$A = B \quad \text{and} \quad AC = BC$$

are equivalent.
That is, each side of the equation may be multiplied by the same nonzero number without changing the solution set.
EXAMPLE 1

Solve \(4x + 8x = -9 + 17x - 1\).

The goal is to isolate \(x\) on one side of the equation.

\[
4x + 8x = -9 + 17x - 1 \\
12x = -10 + 17x \\
12x - 17x = -10 + 17x - 17x \\
-5x = -10 \\
\frac{-5x}{-5} = \frac{-10}{-5} \\
x = 2
\]

Check \(x = 2\) in the original equation.
continued

Check $x = 2$ in the original equation.

\[ 4x + 8x = -9 + 17x - 1 \]
\[ 4(2) + 8(2) = -9 + 17(2) - 1 \]
\[ 8 + 16 = -9 + 34 - 1 \]
\[ 24 = 24 \]

Use parentheses around substituted values to avoid errors.

The true statement indicates that \{2\} is the solution set.

This is NOT the solution.
### Solving a Linear Equation in One Variable

**Step 1**  **Clear fractions.** Eliminate any fractions by multiplying each side by the least common denominator.

**Step 2**  **Simplify each side separately.** Use the distributive property to clear parentheses and combine like terms as needed.

**Step 3**  **Isolate the variable terms on one side.** Use the addition property to get all terms with variables on one side of the equation and all numbers on the other.

**Step 4**  **Isolate the variable.** Use the multiplication property to get an equation with just the variable (with coefficient 1) on one side.

**Step 5**  **Check.** Substitute the proposed solution into the original equation.
Objective 3

Solve linear equations by using the distributive property.
EXAMPLE 2

Solve $6 - (4 + m) = 8m - 2(3m + 5)$.

**Step 1** Since there are no fractions in the equation, Step 1 does not apply.

**Step 2** Use the distributive property to simplify and combine like terms on the left and right.

\[
6 - (4 + m) = 8m - 2(3m + 5)
\]

\[
6 - (1)4 - (1)m = 8m - 2(3m) + (-2)(5)
\]

\[
6 - 4 - m = 8m - 6m - 10
\]
continued

\[ 6 - 4 - m = 8m - 6m - 10 \]

\[ 2 - m = 2m - 10 \]

**Step 3** Next, use the addition property of equality.

\[ 2 - 2 - m = 2m - 10 - 2 \]

\[ -m = 2m - 12 \]

\[ -m - 2m = 2m - 2m - 12 \]

\[ -3m = -12 \]

**Step 4** Use the multiplication property of equality to isolate \( m \) on the left side.

\[ -3m = -12 \]

\[ \frac{-3m}{-3} = \frac{-12}{-3} \]

\[ m = 4 \]
continued

**Step 5** Check: \(6 - (4 + m) = 8m - 2(3m + 5)\)

\[
6 - (4 + 4) = 8(4) - 2(3(4) + 5) \\
6 - 8 = 32 - 2(12 + 5) \\
-2 = 32 - 2(17) \\
-2 = 32 - 34 \\
-2 = -2 \quad \text{True}
\]

The solution checks, so \(\{4\}\) is the solution set.
Objective 4

Solve linear equations with fractions and decimals.
EXAMPLE 3

Solve \( \frac{k+1}{2} + \frac{k+3}{4} = \frac{1}{2} \).

Start by eliminating the fractions. Multiply both sides by the LCD, 4.

**Step 1**

\[
4 \left( \frac{k+1}{2} + \frac{k+3}{4} \right) = 4 \left( \frac{1}{2} \right)
\]

**Step 2**

\[
4 \left( \frac{k+1}{2} \right) + 4 \left( \frac{k+3}{4} \right) = 4 \left( \frac{1}{2} \right) \quad \text{Distributive property.}
\]

\[
\frac{4(k+1)}{2} + \frac{4(k+3)}{4} = 2 \quad \text{Multiply; 4.}
\]
continued

\[
\frac{4(k + 1)}{2} + \frac{4(k + 3)}{4} = 2
\]

\[2(k + 1) + k + 3 = 2\]

\[2(k) + 2(1) + k + 3 = 2\]

Distributive property.

\[2k + 2 + k + 3 = 2\]

Multiply; 4.

\[3k + 5 = 2\]

Combine like terms.

\[3k + 5 - 5 = 2 - 5\]

Subtract 5.

\[3k = -3\]

Combine like terms.

\[\frac{3k}{3} = \frac{-3}{3}\]

Divide by 3.

\[k = -1\]
continued

**Step 5**

Check: \[
\frac{(k + 1)}{2} + \frac{(k + 3)}{4} = \frac{1}{2}
\]

\[
\frac{(k + 1)}{2} + \frac{(k + 3)}{4} = \frac{1}{2}
\]

\[
\frac{(-1+1)}{2} + \frac{(-1+3)}{4} = \frac{1}{2}
\]

\[
0 + \frac{2}{4} = \frac{1}{2}
\]

\[
\frac{1}{2} = \frac{1}{2}
\]

The solution checks, so the solution set is \{-1\}.
EXAMPLE 4

Solve \(0.02(60) + 0.04p = 0.03(50 + p)\).

\[
0.02(60) + 0.04p = 0.03(50 + p) \\
2(60) + 4p = 3(50 + p) \\
120 + 4p = 150 + 3p \\
120 - 120 + 4p = 150 - 120 + 3p \\
4p = 30 + 3p \\
4p - 3p = 30 + 3p - 3p \\
p = 30
\]

Since each decimal number is given in hundredths, multiply both sides of the equation by 100.
Objective 5

Identify conditional equations, contradictions, and identities.
<table>
<thead>
<tr>
<th>Type of Linear Equation</th>
<th>Number of Solutions</th>
<th>Indication when Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional</td>
<td>One</td>
<td>Final line is $x = a$ number.</td>
</tr>
<tr>
<td>Identity</td>
<td>Infinite; solution set ${\text{all real numbers}}$</td>
<td>Final line is true, such as $0 = 0$.</td>
</tr>
<tr>
<td>Contradiction</td>
<td>None; solution set $\emptyset$</td>
<td>Final line is false, such as $-15 = -20$.</td>
</tr>
</tbody>
</table>
EXAMPLE 5

Solve each equation. Decide whether it is a *conditional equation*, an *identity*, or a *contradiction*.

a. \[5(x + 2) - 2(x + 1) = 3x + 1\]
   \[5x + 10 - 2x - 2 = 3x + 1\]
   \[3x + 8 = 3x + 1\]
   \[3x - 3x + 8 = 3x - 3x + 1\]
   \[8 = 1\] \hspace{1cm} \text{False}

The result is false, the equation has no solution. The equation is a contradiction. The solution set is \(\emptyset\).
Solve each equation. Decide whether it is a conditional equation, an identity, or a contradiction.

\[ \frac{x+1}{3} + \frac{2x}{3} = x + \frac{1}{3} \]

Multiply each side by the LCD, 3.

\[ 3 \left( \frac{x+1}{3} \right) + 3 \left( \frac{2x}{3} \right) = 3 \left( x + \frac{1}{3} \right) \]

\[ x+1+2x = 3x+1 \]

\[ 3x+1 = 3x+1 \]

This is an identity. Any real number will make the equation true. The solution set is \{all real numbers\}. 

continued
continued

Solve each equation. Decide whether it is a *conditional equation*, an *identity*, or a *contradiction*.

c. \[5(3x + 1) = x + 5\]

\[15x + 5 = x + 5\]

\[15x - x + 5 = x - x + 5\]

\[14x + 5 = 5\]

\[14x + 5 - 5 = 5 - 5\]

\[14x = 0\]

\[x = 0\]

This is a conditional equation. The solution set is \{0\}. 
Formulas

1. Solve a formula for a specified variable.
2. Solve applied problems by using formulas.
3. Solve percent problems.
Objective 1

Solve a formula for a specified variable.
A **mathematical model** is an equation or inequality that describes a real situation. Models for many applied problems already exist; they are called *formulas*. A **formula** is the an equation in which variables are used to describe a relationship.

Some formulas are

\[ d = rt, \quad I = p rt, \quad \text{and} \quad P = 2L + 2W. \]
Solving for a Specified Variable

**Step 1** Transform so that all terms containing the specified variable are on one side of the equation and all terms without that variable are on the other side.

**Step 2** If necessary, use the distributive property to combine the terms with the specified variable.* The result should be the product of the sum or difference and the variable.

**Step 3** Divide both sides by the factor that is the coefficient of the specified variable.
EXAMPLE 1

Solve \( m = 2k + 3b \) for \( k \).

Solve the formula by isolating the \( k \) on one side of the equals sign.

\[
m = 2k + 3b
\]

**Step 1**

\[
m - 3b = 2k + 3b - 3b
\]

\[
m - 3b = 2k
\]

**Step 2**

\[
\frac{m - 3b}{2} = \frac{2k}{2}
\]

**Step 3**

\[
\frac{m - 3b}{2} = k \quad \text{or} \quad k = \frac{m - 3b}{2}
\]
EXAMPLE 2

Solve the formula \( y = \frac{1}{2} x + 3 \) for \( x \).

\[
y = \frac{1}{2} x + 3
\]

\[
2y = x + 3 \quad \text{Multiply by 2.}
\]

\[
2y - 3 = x \quad \text{or} \quad x = 2y - 3 \quad \text{Subtract 3.}
\]
EXAMPLE 3

Solve the formula $A = 2HW + 2LW + 2LH$ for $W$.

\[
A = 2HW + 2LW + 2LH
\]

\[
A - 2LH = 2HW + 2LW \quad \text{Subtract 2LH.}
\]

\[
A - 2LH = W(2H + 2L) \quad \text{Distributive property}
\]

\[
\frac{A - 2LH}{2H + 2L} = \frac{W \cdot (2H + 2L)}{2H + 2L}
\]

\[
\frac{A - 2LH}{2H + 2L} = W, \quad \text{or} \quad W = \frac{A - 2LH}{2H + 2L}
\]
CAUTION  The most common error in working a problem like in Example 3 is not using the distributive property correctly. We must write the expression so that the specified variable is a *factor*; then we can divide by its coefficient in the final step.
Objective 2

Solve applied problems by using formulas.
EXAMPLE 4

The distance is 500 mi and the rate is 25 mph. Find the time.

Find the formula for time by solving \( d = rt \) for \( t \).

\[
d = rt
\]

\[
\frac{d}{r} = \frac{rt}{r} \quad \text{Divide by } r.
\]

\[
\frac{d}{r} = t \quad \text{or} \quad t = \frac{d}{r}
\]
Now substitute $d = 500$ and $r = 25$.

$$t = \frac{d}{r}$$

$$t = \frac{500}{25}$$

Let $d = 500$, $r = 25$.

$$t = 20$$

Divide.

The time is 20 hours.
**PROBLEM-SOLVING HINT** As seen in Example 4, it may be convenient to first solve for a specified unknown variable before substituting the given values. This is particularly useful when we wish to substitute several different values for the same variable. For example, an economics class might need to solve the equation $I = prt$ for $r$ to find rates that produce specified amounts of interest for various principals and times.
Objective 3

Solve percent problems.
An important everyday use of mathematics involves the concept of percent. Percent is written with the symbol %. The word percent means “per one hundred”.

\[ 1\% = 0.01 \quad \text{or} \quad 1\% = \frac{1}{100} \]

The following formula can be used to solve a percent problem:

\[ \frac{\text{partial amount}}{\text{whole amount}} = \text{percent} \] (represented as a decimal).
EXAMPLE 5

Solve each problem.

a. A mixture of gasoline oil contains 20 oz, of which 1 oz is oil. What percent of the mixture is oil?

The given amount of mixture is 20 oz. The part that is oil is 1 oz. Thus, the percent of oil is

\[
x = \frac{1}{20}
\]

\[x = 0.05, \text{ or } 5\%.
\]

Thus, 5\% of the mixture is oil.
b. An automobile salesman earns an 8% commission on every car he sells. How much does he earn on a car that sells for $12,000?

Let \( x \) represent the amount of commission earned.

\[
8\% = 8 \cdot 0.01 = 0.08
\]

\[
\frac{x}{12,000} = 0.08
\]

\[
x = 0.08 \times 12,000
\]

\[
x = 960
\]

The salesman earns $960.
EXAMPLE 6

In 2005, people in the United States spent an estimated $35.9 billion on their pets. How much was spent on pet supplies/medicine? Round your answer to the nearest tenth of a billion dollars.

Let $x$ represent the amount spent on pet supplies/medicine.

\[
\frac{x}{35.9} = 0.245
\]

Therefore, about $8.8 billion was spent on pet supplies/medicine.

\[
x = 8.7955
\]
Applications of Linear Equations

1. Translate from words to mathematical expressions.
2. Write equations from given information.
3. Distinguish between expressions and equations.
4. Use the six steps in solving an applied problem.
5. Solve percent problems.
7. Solve mixture problems.
Objective 1

Translate from words to mathematical expressions.
<table>
<thead>
<tr>
<th>Verbal Expression</th>
<th>Mathematical Expression (where x and y are numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td></td>
</tr>
<tr>
<td>The sum of a number and 7</td>
<td>$x + 7$</td>
</tr>
<tr>
<td>6 more than a number</td>
<td>$x + 6$</td>
</tr>
<tr>
<td>3 plus a number</td>
<td>$3 + x$</td>
</tr>
<tr>
<td>24 added to a number</td>
<td>$x + 24$</td>
</tr>
<tr>
<td>A number increased by 5</td>
<td>$x + 5$</td>
</tr>
<tr>
<td>The sum of two numbers</td>
<td>$x + y$</td>
</tr>
<tr>
<td><strong>Verbal Expression</strong></td>
<td><strong>Mathematical Expression</strong> (where ( x ) and ( y ) are numbers)</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td></td>
</tr>
<tr>
<td>2 less than a number</td>
<td>( x - 2 )</td>
</tr>
<tr>
<td>2 less a number</td>
<td>( 2 - x )</td>
</tr>
<tr>
<td>12 minus a number</td>
<td>( 12 - x )</td>
</tr>
<tr>
<td>A number decreased by 12</td>
<td>( x - 12 )</td>
</tr>
<tr>
<td>A number subtracted from 10</td>
<td>( 10 - x )</td>
</tr>
<tr>
<td>From a number, subtract 10</td>
<td>( x - 10 )</td>
</tr>
<tr>
<td>The difference between two numbers</td>
<td>( x - y )</td>
</tr>
<tr>
<td><strong>Verbal Expression</strong></td>
<td><strong>Mathematical Expression</strong></td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td></td>
</tr>
<tr>
<td>16 times a number</td>
<td>$16x$</td>
</tr>
<tr>
<td>A number multiplied by 6</td>
<td>$6x$</td>
</tr>
<tr>
<td>2/3 of a number</td>
<td>$\frac{2}{3}x$</td>
</tr>
<tr>
<td>¾ as much as a number</td>
<td>$\frac{3}{4}x$</td>
</tr>
<tr>
<td>Twice (2 times) a number</td>
<td>$2x$</td>
</tr>
<tr>
<td>The product of two numbers</td>
<td>$xy$</td>
</tr>
<tr>
<td>Verbal Expression</td>
<td>Mathematical Expression (where $x$ and $y$ are numbers)</td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
<td>---------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td></td>
</tr>
<tr>
<td>The quotient of 8 and a number</td>
<td>$\frac{8}{x}$ ($x \neq 0$)</td>
</tr>
<tr>
<td>A number <strong>divided by</strong> 13</td>
<td>$\frac{x}{13}$</td>
</tr>
<tr>
<td>The <strong>ratio</strong> of two numbers or the <strong>quotient</strong> of two numbers</td>
<td>$\frac{x}{y}$ ($y \neq 0$)</td>
</tr>
</tbody>
</table>
Objective

2

Write equations from given information.
EXAMPLE 1

Translate each verbal sentence into an equation, using $x$ as the variable.

a. The sum of a number and 6 is 28.  \[ x + 6 = 28 \]

b. The product of a number and 7 is twice the number plus 12.  \[ 7x = 2x + 12 \]

c. The quotient of a number and 6, added to twice the number is 7.  \[ 2x + \frac{x}{6} = 7 \]
Objective 3

Distinguish between expressions and equations.
EXAMPLE 2

Decide whether each is an *expression* or *equation*.

a. \(5x - 3(x + 2) = 7\)

   There is an equals sign with something on either side of it, this is an equation.

b. \(5x - 3(x + 2)\)

   There is no equals sign, so this is an expression.
Objective 4

Use the six steps in solving an applied problem.
Solving an Applied Problem

**Step 1** Read the problem, several times if necessary, until you understand what is given and what is to be found.

**Step 2** Assign a variable to represent the unknown value, using diagrams or tables as needed. Write down what the variable represents. If necessary, express any other unknown values in terms of the variable.

**Step 3** Write an equation using the variable expression(s).

**Step 4** Solve the equation.

**Step 5** State the answer to the problem. Does it seem reasonable?

**Step 6** Check the answer in the words of the original problem.
EXAMPLE 3

The length of a rectangle is 5 cm more than its width. The perimeter is five times the width. What are the dimensions of the rectangle?

Step 1  Read the problem. What must be found?

The length and width of the rectangle.

What is given?

The length is 5 cm more than its width; the perimeter is 5 times the width.

Step 2  Assign a variable. Let \( w = \) the width; then \( w + 5 = \) the length. Make a sketch.
continued

**Step 3  Write an equation.** Use the formula for the perimeter of a rectangle.

\[ P = 2l + 2w \]

\[ 5w = 2(w + 5) + 2(w) \]

**Step 4  Solve** the equation.

\[ 5w = 2w + 10 + 2w \]

\[ 5w = 4w + 10 \]

\[ 5w - 4w = 4w - 4w + 10 \]

\[ w = 10 \]
Step 5 State the answer. The width of the rectangle is 10 cm and the length is $10 + 5 = 15$ cm.

Step 6 Check.

The perimeter is 5 times the width.

$$P = 2l + 2w$$

$$5w = 2(15) + 2(10)$$

$$50 = 30 + 20$$

$$50 = 50$$

The solution checks.
EXAMPLE 4

For the 2005 baseball season, the Major League Baseball leaders in runs batted in (RBIs) were Andruw Jones of the Atlanta Braves in the National League and David Ortiz of the Boston Red Sox in the American League. These two players had a total of 276 RBIs, and Ortiz had 20 more than Jones. How many RBIs did each player have?

Step 1 Read the problem. We are asked to find the number of RBIs each player had.
continued

**Step 2 Assign a variable.**

Let $x$ represent the number of RBIs for Jones.
Let $x + 20$ represent the RBIs for Ortiz

**Step 3 Write an equation.**

The sum of the RBI’s is 276.

$x + x + 20 = 276$

**Step 4 Solve the equation.**

$x + x + 20 = 276$

$2x + 20 = 276$

$2x + 20 - 20 = 276 - 20$

$2x = 256$

$x = 128$
Step 4 \[ x = 128 \]

Step 5 State the answer. We let \( x \) represent the number of RBIs for Jones.

Then Ortiz has \( x + 20 = 128 + 20 = 148 \)

Step 6 Check.

148 is 20 more than 128, and 148 + 128 = 276. The conditions of the problem are satisfied, and our answer checks.
Objective 5

Solve percent problems.
EXAMPLE 5

187.5 is 125% of some number. What is that number?

**Step 1  Read** the problem. We are given 187.5 is 125% of some number. We are asked to find the number.

**Step 2  Assign a variable.**

Let \( x \) = the number

125\% = 1.25

**Step 3  Write an equation** from the given information.

187.5 is 125\% of some number

187.5 = 1.25x
continued

**Step 4  Solve** the equation.

\[ 187.5 = 1.25x \]

\[
\begin{align*}
\frac{187.5}{1.25} &= \frac{1.25x}{1.25} \\
150 &= x
\end{align*}
\]

**Step 5  State the answer.** 187.5 is 125\% of 150.

**Step 6  Check.**

\[ 150 \cdot 125\% = 187.5 \] (the answer checks)
Objective 6

Solve investment problems.
EXAMPLE 6

A man has $34,000 to invest. He invests some of the money at 5% and the balance at 4%. His total annual interest income is $1545. Find the amount invested at each rate.

Step 1  Read the problem. We must find the two amounts.

Step 2  Assign a variable.

Let $x = \text{the amount to invest at 5%}$

$34,000 - x = \text{the amount to invest at 4%}$
continued

**Step 2** Assign a variable.

The formula for interest is $I = prt$. Here the time is 1 yr. Use a table to organize the given information.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate (as a decimal)</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>0.05</td>
<td>0.05$x$</td>
</tr>
<tr>
<td>$34,000 - x$</td>
<td>0.04</td>
<td>0.04($34,000 - x$)</td>
</tr>
<tr>
<td>$34,000$</td>
<td>XXXXXXXXXXXX</td>
<td>1545</td>
</tr>
</tbody>
</table>

**Step 3** Write an equation. The last column of the table gives the equation.

interest at 5% + interest at 4% = total interest

\[0.05x + 0.04(34,000 - x) = 1545]
Step 4 Solve the equation. We do so without clearing decimals.

\[ 0.05x + 0.04(34,000 - x) = 1545 \]

\[ 0.05x + 1360 - 0.04x = 1545 \]

\[ 0.01x + 1360 = 1545 \]

\[ 0.01x = 185 \]

\[ x = 18,500 \]

Step 5 State the answer. $18,500 was invested at 5% and 15,500 was invested at 4%.

Step 6 Check by finding the annual interest at each rate.

\[ 0.05(18,500) = 925 \]

\[ 0.04(15,500) = 620 \]

\[ 925 + 620 = 1545 \]
Objective 7

Solve mixture problems.
EXAMPLE 7

How many pounds of candy worth $8 per lb should be mixed with 100 lb of candy worth $4 per lb to get a mixture that can be sold for $7 per lb?

**Step 1  Read** the problem. The problem asks for the amount of candy worth $8 to be used.

**Step 2  Assign a variable.** Let $x = \text{the amount of$8 candy}

<table>
<thead>
<tr>
<th>Number of pounds</th>
<th>$ Amount</th>
<th>Pounds of Candy worth $7</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$4</td>
<td>100(4) = 400</td>
</tr>
<tr>
<td>$x</td>
<td>$8</td>
<td>8$x$</td>
</tr>
<tr>
<td>100 + $x$</td>
<td>$7</td>
<td>7(100 + x)</td>
</tr>
</tbody>
</table>

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continued

**Step 3** Write an equation.

\[ 400 + 8x = 7(100 + x) \]

**Step 4** Solve.

\[ 400 + 8x = 700 + 7x \]

\[ x = 300 \]

**Step 5** State the answer. 300 pounds of candy worth $8 per pound should be used.

**Step 6** Check.

300 lb worth $8 + 100 lb worth $4 = $7(100 + 300)

\[ $2400 + $400 = $7(400) \]

\[ $2800 = $2800 \]
EXAMPLE 8

How much water must be added to 20 L of 50% antifreeze solution to reduce it to 40% antifreeze?

**Step 1** Read the problem. The problem asks for the amount of pure water to be added.

**Step 2** Assign a variable. Let $x =$ the number of liters of pure water

<table>
<thead>
<tr>
<th>Number of liters</th>
<th>Percent (as a decimal)</th>
<th>Liters of Pure Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
<td>20(0.5)</td>
</tr>
<tr>
<td>$x + 20$</td>
<td>0.4</td>
<td>0.4($x + 20$)</td>
</tr>
</tbody>
</table>
continued

Step 3  Write an equation.

\[ 0 + 20(0.5) = 0.4(x + 20) \]

Step 4  Solve.

\[ 10 = 0.4x + 8 \]
\[ 2 = 0.4x \]
\[ x = 5 \]

Step 5  State the answer.  5 L of water are needed.

Step 6  Check.

\[ 20(0.5) = 0.4(5 + 20) \]
\[ 10 = 0.4(25) \]
\[ 10 = 10 \]
Further Applications of Linear Equations

1. Solve problems about different denominations of money.
2. Solve problems about uniform motion.
3. Solve problems about angles.
Objective 1

Solve problems about different denominations of money.
PROBLEM-SOLVING HINT

In problems involving money, use the basic fact that

\[
\text{Number of monetary units of the same kind} \times \text{denomination} = \text{total monetary value}.
\]

For example, 30 dimes have a monetary value of 30\((\$0.10) = \$3.00\). Fifteen 5-dollar bills have a value of 15\((\$5) = \$75\).
EXAMPLE 1

Mohammed has a box of coins containing only three dimes and half-dollars. There are 26 coins, and the total value is $8.60. How many of each denomination of coin does he have?

Step 1 Read the problem. The problem asks that we find the number of each denomination of coin that Mohammed has.

Step 2 Assign a variable.
Let \( x \) = the number of dimes
Let \( 26 - x \) = number of half-dollars.
Multiply the number of coins by the denominations, and add the results to get 8.60.

Step 3  Write an equation.

\[0.10x + 0.50(26 - x) = 8.60.\]
Step 4  Solve.

\[ 0.10x + 0.50(26 - x) = 8.60 \quad \text{Multiply by 10.} \]

\[ 1x + 5(26 - x) = 86 \quad \text{Distributive property.} \]

\[ 1x + 130 - 5x = 86 \]

\[ -4x = -44 \]

\[ x = 11 \]

Step 5  State the answer. He has 11 dimes and 26 – 11 = 15 half-dollars.

Step 6  Check. He has 11 + 15 = 26 coins, and the value is $0.10(11) + $0.50(15) = $8.60.
Objective 2

Solve problems about uniform motion.
PROBLEM-SOLVING HINT

Uniform motion problems use the distance formula, \( d = rt \). In this formula, \textit{when rate (or speed) is given in miles per hour, time must be given in hours. To solve such problems, draw a sketch} to illustrate what is happening in the problem, and \textit{make a table} to summarize the given information,
EXAMPLE 2

Two cars leave the same town at the same time. One travels north at 60 mph and the other south at 45 mph. In how many hours will they be 420 mi apart?

Step 1  Read the problem. We are looking for the time that it takes for the cars to be 420 miles apart.

Step 2  Assign a variable. A sketch shows what is happening.
Let $x$ = the amount of time needed for the cars to be 420 mi apart.

<table>
<thead>
<tr>
<th></th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Northbound Car</strong></td>
<td>60</td>
<td>$x$</td>
<td>$60x$</td>
</tr>
<tr>
<td><strong>Southbound Car</strong></td>
<td>45</td>
<td>$x$</td>
<td>$45x$</td>
</tr>
<tr>
<td><strong>XXX XXX XXX XXX</strong></td>
<td><strong>XXX</strong></td>
<td><strong>XXX</strong></td>
<td><strong>420</strong></td>
</tr>
</tbody>
</table>

**Step 3** Write an equation.

$$60x + 45x = 420$$
continued

Step 4  Solve.  \[60x + 45x = 420\]
\[105x = 420\]
\[x = \frac{420}{105} = 4\]

Step 5  State the answer. The cars will be 420 mi apart in 4 hr.

Step 6  Check.  \[60(4) + 45(4) = 420\]
\[240 + 180 = 420\]
\[420 = 420\]
CAUTION  It is a common error to write 420 as the distance traveled by each car in Example 2. Four hundred twenty is the *total* distance traveled.

\[
\text{partial distance} + \text{partial distance} = \text{total distance}.
\]
EXAMPLE 3

When Michael drives his car to work, the trip takes $\frac{1}{2}$ hr. When he rides the bus, it takes $\frac{3}{4}$ hr. The speed of the bus is 12 mph less than the speed when driving his car. Find the distance he travels to work.

**Step 1** Read the problem. We are looking for the distance Michael travels to his workplace.

**Step 2** Assign a variable.
Let $x =$ the speed.
Then $x - 12 =$ the speed of the bus.
Step 3  Write an equation.  \( \frac{1}{2} x = \frac{3}{4} (x - 12) \)

Step 4  Solve.  \( \frac{1}{2} x = \frac{3}{4} x - 12 \)

\[
\begin{align*}
2x & = 3x - 36 \\
36 & = x
\end{align*}
\]
**Step 5** State the answer.

The required distance is

\[ d = \frac{1}{2} x = \frac{1}{2} \times 36 = 18 \text{ miles.} \]

**Step 6** Check.

\[ d = \frac{3}{4} x - 12 \]

\[ d = \frac{3}{4} \times 36 - 12 \]

\[ d = \frac{3}{4} \times 24 \]

\[ d = 18 \text{ miles} \]

Same result.
Objective 3

Solve problems about angles.
EXAMPLE 4

Find the value of $x$, and determine the measure of each angle.

**Step 1  Read** the problem. We are asked to find the measure of each angle.

**Step 2  Assign a variable.**

Let $x = \text{the measure of one angle.}$
continued

**Step 3** Write an equation. The sum of the three measures shown in the figure must be 180°.

\[ x + x + 61 + 2x + 7 = 180 \]

**Step 4** Solve.

\[ 4x + 68 = 180 \]

\[ 4x = 112 \]

\[ x = 28 \]

**Step 5** State the answer. The angles measure 28°, 28 + 61 = 89°, and 2(28) + 7 = 63°.

**Step 6** Check. 28° + 89° + 63° = 180°.